Abstract Week 5

Symplectic Geometry, Gauge Theory, and Low-Dimensional Topology Seminar

Speaker: Egor Shelukhin

Abstract: The growth of an autonomous Hamiltonian flow in Hofer's metric is not yet well understood. A result of Polterovich and Rosen shows that generically this growth is asymptotically linear, and in all known cases where it is not, it appears to be bounded. Such a dichotomy is known for open connected surfaces of infinite area by a result of Polterovich and Siburg from 2000. I will discuss a new approach to this question, which establishes a strong version of such a dichotomy for the two-sphere. This talk is based on a joint work with Lev Buhovsky, Ben Feuerstein, and Leonid Polterovich.

Mini Course:

Speaker: Egor Shelukhin

Abstract: We discuss a metric on the identity component of the group of contactomorphisms induced by the contact Hamiltonian construction. The definition depends on the choice of a contact form and therefore produces a metric which is right invariant but not bi-invariant, in contrast to Hofer's celebrated metric on the group of Hamiltonian diffeomorphisms of a symplectic manifold. Nevertheless, this metric and its version for the universal cover of the group of contactomorphisms have interesting properties, which allow one to detect invariant features of the contact structure. In particular, I will discuss a joint work with Jakob Hedicke detecting the non-orderability of a contact manifold in terms of shortening in contact Hofer geometry.

Speaker: Erasmo Caponio

Abstract: Kropina and Randers-Kropina metrics constitute a class of singular Finsler metrics that naturally arise in both physical and geometric models. They describe the projection on a spacelike slice of lightlike vectors in spacetimes endowed with a causal Killing vector field. Moreover, their geodesics are solutions to Zermelo's navigation problem when a ship moves under the influence of a time-independent wind represented by a vector field whose norm, with respect to a background Riemannian metric, does not exceed one. The singular, Kropina, case corresponds to points where the wind has unit norm or, equivalently in the spacetime picture, where the Killing field is lightlike. At such points, these metrics fail to be continuously extendible at the zero vector and on the kernel of their associated one-form, posing significant challenges to the standard variational approaches for finding geodesics.

In this talk, we will explore the existence of connecting and closed geodesics in these spaces. To overcome the singular nature of the Lagrangian, we introduce an approximation scheme that perturbs the Randers-Kropina metric using a family of regular Randers metrics. Multiplicity results are then obtained by combining Lusternik-Schnirelman theory for the approximating

functionals with a recent result on the homotopy type of the space of solutions of a differential inclusion defined by a field of open half-spaces associated with a completely nonholonomic corank one distribution of step 2. Finally, we will comment on the open problem of finding one closed geodesic in the simply connected compact case.

Speakers: Jakob Hedicke and Stefan Suhr

Abstract: In the first half of this mini course we will review the constructions of the natural topology, smooth structure and contact structure on the space of all future pointing null geodesics of a Lorentzian manifold. We will discuss various known criteria that ensure that the space of null geodesics is a smooth manifold. Here the main focus lies on the case of causally simple spacetimes, where the topology on space of null geodesics can be connected to the conformal extendibility of the spacetime.

In the second half we will discuss examples of compact spacetimes with a smooth space of null geodesics ("Zollfrei") and the important class of Kerr* spacetimes. For the later the space of null geodesics is known not to be Hausdorff. But it is an open problem whether it can be turned into a Hausdorff space by a finite quotient contraction. We will discuss indications for this.