Beyond Λ: extracting Dark energy information in an era of tensions

Why these notes:

↳ use of toy models (I LOVE toy models) so you can analytically understand more on what is being discussed in cosmology (H_0 & DESI tensions)

• Except for the Hu & Sugiyama CMB solution, everything here can be understood with basic cosmology knowledge. I tried to be didactical (sorry if I failed)

↳ I can't present the entire calculation in 90 mins, but I hope the notes will inspire you!

I wrote part of the notes during my Jamaica-SBU train rides, so I apologize for shaky handwrite.

What is missing? S_8 tension.

Cosmology cheat sheet (101)

- $W(z)$CDM : Radiation + Dark Matter + Baryons + Dark Energy

- Comoving distance: $D(z)$.

- We will assume flatness

- Define $Q(z) \equiv H(z)/100\,\mathrm{Km/s/Mpc}$, $h \equiv \dfrac{H_0}{100\,\mathrm{Km/s/Mpc}}$

Then,

$$Q^2(z) = \Omega_R h^2 (1+z)^4 + \Omega_m h^2 (1+z)^3 + (h^2 - \Omega_m h^2 - \Omega_R h^2) f(z)$$

with $f(z) = 1$ for Λ ; $f(z) = \exp\left[3 \int_0^z d\ln(1+z')(1+w(z'))\right]$

- Now $D(z) = \dfrac{c\,[\mathrm{km/s}]}{100\,\mathrm{Km/s/Mpc}} \displaystyle\int_0^z \frac{dz'}{Q(z')}$. Sometimes also

useful to write as $\dfrac{c}{H_0} \displaystyle\int_0^z \frac{dz'}{E(z')}$ with $E(z) = H(z)/H_0$

- $\Omega_m h^2 = \underbrace{\Omega_c h^2}_{\text{dark matter}} + \underbrace{\Omega_b h^2}_{\text{Baryons}} + \underbrace{\Omega_\nu h^2}_{\text{neutrinos}}$ and $\Omega_\nu h^2 \sim \dfrac{m_\nu}{94.1\,\mathrm{eV}}$.

$\underbrace{\Omega_c h^2 + \Omega_b h^2}_{} \rightarrow \Omega_{cb} h^2$

- Matter-radiation equality

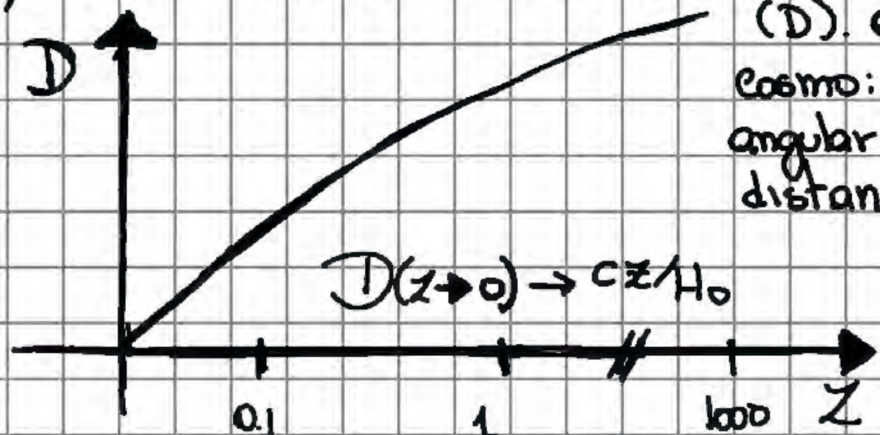
$$\frac{\rho_m}{\rho_r} \approx 24\,\Omega_m h^2 \left(\frac{1000}{1+z}\right) \; ; \quad \Omega_m h^2 \approx 0.14$$

$$\frac{\rho_m}{\rho_r} \sim \left(\frac{1000}{1+z}\right) 3.36$$

- Main topics
  (1) What is the H_0 tension & why late-time dark energy fails to solve it
  (2) How Early dark energy solves the H_0 tension but it creates changes you don't expect on $n_s / \Omega_m h^2$.
  (3) How CMB + BAO measures Σm_ν and how that depends on dark energy.
  (4) Why τ is a hot topic in cosmo? (optical depth)

Cosmology has "many" distances: D_L, D_A, D...

Here, I will convert them all to comoving distance (D). On flat cosmo: comoving angular diameter distance $D_A = D$



$D\left(z \mid H_0, \text{Dark matter}, \text{Dark energy}, \text{Radiation}, \text{curvature}, \ldots\right)$

→ complicated shape: you may think we must seek the $z \to 0$ to measure H_0. False! If you know the cosmology (except for H_0), ie, we know the z evolution of $D(z)$, the measurements of $D(z)$ at any z allows us to infer H_0

Example: recombination redshift $z_* \sim 1100$.

↳ you may wonder how can we determine observationally the evolution of $D(z)$ without getting any information on H_0.

↳ it turns out in astronomy it is a lot easier to determine relative distances than absolute distances (e.g., type Ia supernovae & cepheids/mira stars). On distance ratios, H_0 drops

↳ CMB & BAO also measure relative distances, but to the sound horizon of accoustic oscillations. We think the universe is simple up to redshift of recombination (early dark energy can change the story) so we can compute from first principles the sound horizon!

↳ this is how the H_0 tension entangle early and late time physics.

Why measuring relative distances is "easier"

→ Cepheids/Mira stars : 2 complicated

type of oscillatory stars:

Mira : $0.8 - 8 M_\odot$, Pulsation Period 80-1000 days

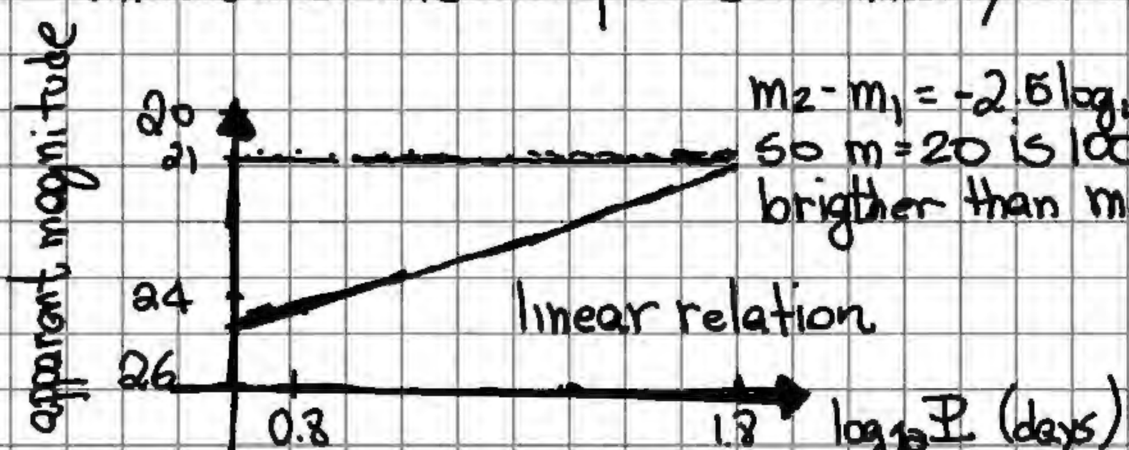
they are red giants. Pulsation amplitude: large (2.5 mag), size: $100-500 R_\odot$, luminosity: $10^3 - 10^4 L_\odot$

Cepheids $3 - 12 M_\odot$, Pulsation Period: 1-100 days

superluminous supergiant. Pulsation amplitude: moderate 0.1-2 mag, Luminosity $10^3 - 10^5 L_\odot$, Size: $30-200 R_\odot$

Mira/Cepheids stars are not all the same. They don't all have the same mass, luminosity, metalicity... So quite hard to determine absolute luminosity

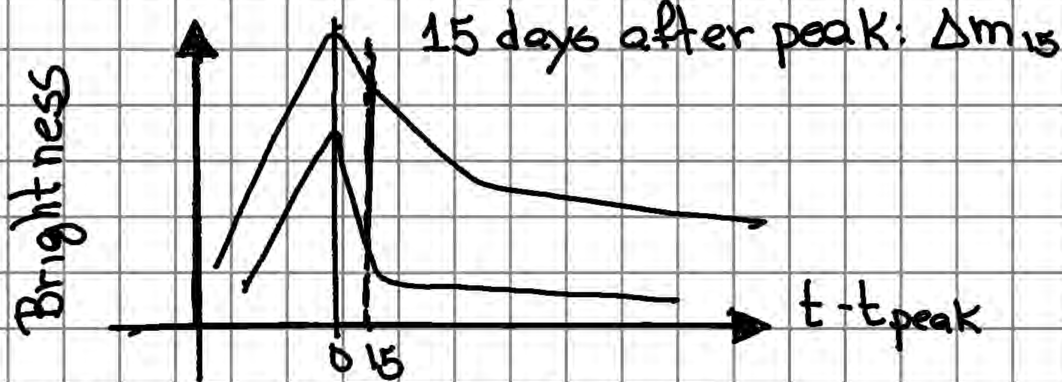
→ But the physics of these oscillations is well understood and the period-luminosity relation



$m_2 - m_1 = -2.5 \log_{10}(F_2/F_1)$

so $m = 20$ is $100\times$ brighter than $m = 25$

So the period luminosity relation allow us to get relative distances between cepheids in the local group ($D \lesssim$ Mpc) and $D \lesssim 40$ Mpc. For $D \gtrsim 40$ Mpc We need Type Ia supernova, which are calibrated via the Phillips relation



The luminosity of more luminous SN decays slower comparing peak day and 15 days after peak (Δm_{15})

→ SN are not all the same, but we can compare their relative distance. Why do we need cepheids?

The good news for life is that SN are rare.

$N_{SN,Ia}(z < 0.01) \sim 80$/decade ; $D(z \sim 0.01) \sim 45$ Mpc

We can't measure parallax up to 45 Mpc!

So we need cepheids to reach $D \sim 40 Mpc$. And finally how we calibrate cepheids, so we can measure H_0? Either <u>Parallax</u> or <u>water masers</u>

(on gas orbits near BH)

→ Parallax: $\underbrace{1}_{\text{arc second}}$ arcsec → D = 1pc when orbit of Earth is the baseline. So to reach 1Mpc, Hubble & Gaia needs to measure changes in position with good enought SNR at the order of $\sim \underbrace{1 \mu as}_{10^{-6} arcsec}$!

So original SHOES was

Parallax → Cepheids → Type Ia SN

$D \ll 1 Mpc$ $\quad$ $D \lesssim 45 Mpc$ $\quad$ $D \gtrsim 45 Mpc$

But we could have also done

BAO ($z \sim 0.5$) + BBN ($\Omega_b h^2$) → Type Ia SN (from $z = 0.5$ to $D \gtrsim 45 Mpc$) → H_0 !

SHOES: direct distance ladder

from BAO: inverse distance ladder (IDL)

- In LCDM, SN is "optional" on IDL if you have BAO on a large z range (to probe $\Omega_m h^2$) $(1+z)^3$ term) + BBN (to probe $\Omega_b h^2$ so r_s can be ping)

  ↳ the SN allows you to compute IDL for a generic $w(z)$ function to see if any Dark energy behaviour in the range $z \leq 0.5$ could explain the Hubble tension (answer is <u>No</u>!) (see section "Why Dark energy fails to solve the H_0 tension")

  ↳ Key is the <u>lack</u> of a $\underset{\text{magnitude}}{\Delta m}$ drift of ~ 0.19 not observed in SN w/ similar Δm_{15} decays (Phillips relation) on $z \sim 0$ vs $z \sim 0.5$ (compared to ΛCDM expectations

Summary of distances

Probe —— z range —— units

- BAO —— $0.3 < z < 2$ —— r_d (drag horizon)
- CMB —— $z \sim 1100$ —— r_s (sound horizon)
- Type Ia SN —— $0.01 \lesssim z \lesssim 2$ —— $\underbrace{D \text{ (anchor)}}$

parallax pin cepheids

- $r_s \sim r_d$ (they differ by 2%)
  - → r_s: sound Horizon. Think τ as $\underbrace{I}_{\text{rad intensity}}(t_2) = I(t_1)\, e^{-\Delta t/\tau}$
  - → drag horizon: Think τ_d as $\Delta v(t_2) = \Delta v(t_1)\, e^{-\Delta t/\tau_d}$ (how much $v_b \neq v_\gamma$)

- the "units" is the source of the dependence of both (1) BAO/CMB H_0 inference on early universe physics (example EDE); (2) SH0ES H_0 inference on a variety of complicated systematics

- Why type Ia SN needs anchors? Answer lies in the Phillips relation

15 days after peak Δm_{15} (m is magnitude)

$\Delta m = -2.5 \log_{10}(\text{Flux ratio})$

$F_2 = 100 F_1 \Rightarrow \Delta m = -5$

so we can compare SN between themselves but we still need an anchor

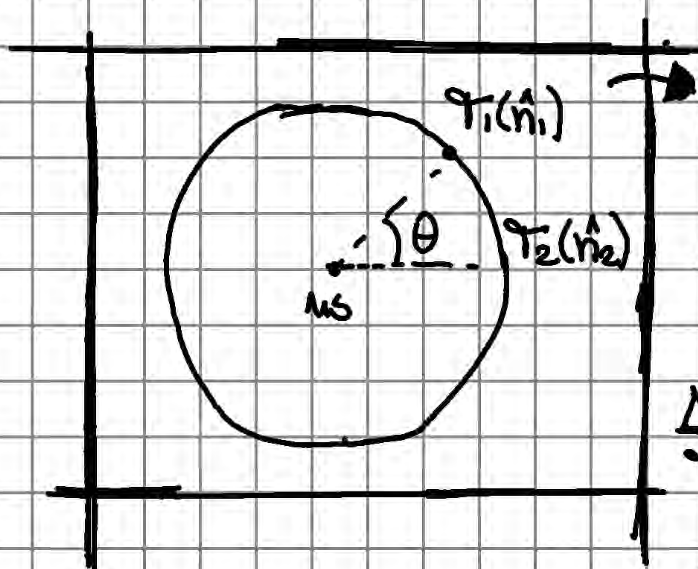
But I want to talk on how the CMB "measures" H_0

- CMB & H_0

↳ CMB   Monopole ~ 2.7K
    anisotropies ~ 10^{-5} $(\Delta T/T)$

- Black body radiation $\frac{\delta \rho_\gamma}{\rho_\gamma} = 4\frac{\Delta T}{T}$

- I will assume instant recombination (Toy model ❢) → CMB then is a spherical projection of the inhomogeneiteis at $z_* \sim 1100$ (recombination)

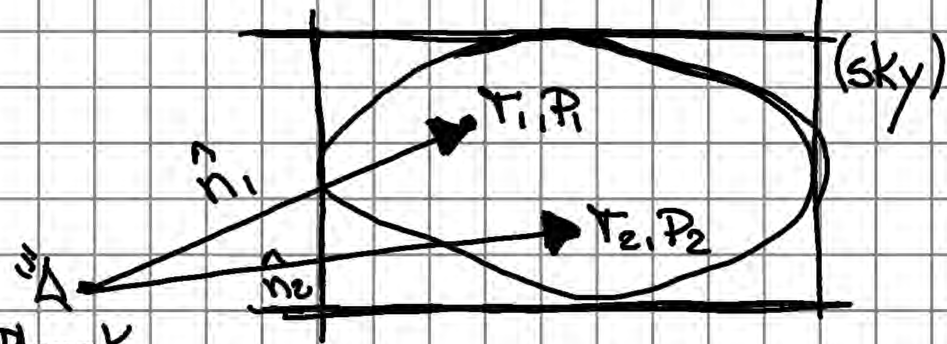


universe at $z_* \sim 1100$

$$\frac{\Delta T(\theta)}{T} \xrightarrow{\text{projection}} \frac{\Delta T}{T}(x)$$

$$\frac{\Delta T(\theta)}{T} \sim \int d^3K \, \frac{\Delta T}{T}(k) \, e^{i\hat{K}\cdot\hat{n}_R D_R}$$

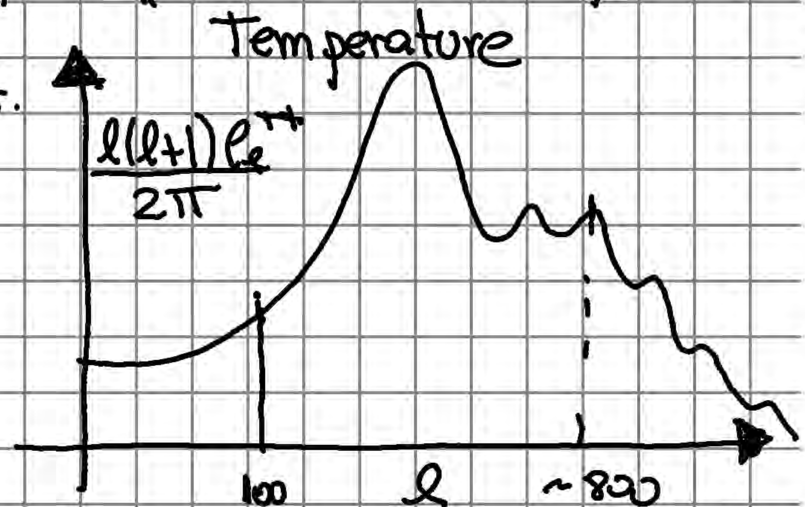
D_R = distance to recombination. We extract information via correlation functions in the sphere



Planck
ACT
SPT

Temperature spectrum $\left\langle \frac{\Delta T}{T}(\hat{n}_1)\, \frac{\Delta T(\hat{n}_2)}{T} \right\rangle$

In Fourie space, that is the famous temperature/polarization spectrum



This looks like a distorted cosine squared

$$\frac{\ell(\ell+1) C_\ell}{2\pi} \sim A(\ell) \cos^2(\ell \theta_*) \quad \text{(Toy model !)}$$

$\boxed{\text{an accoustic angular horizon}}$

where $\theta_* \sim \frac{r_s}{D(z_*)}$, z_* distance to recombinations

and r_s the accoustic horizon $\left(\frac{\Delta T}{T}\right)(\hat{k}) \sim \cos(K r_s)$

in real space (comoving coordinates)

That is remarkable, as it shows:

(I) The physics of each K mode is the physics of an harmonic oscillator with a particular set of initial conditions

(II) different K's have a <u>coherent phase</u>

(II) is not always appreciated: an active source that drives, across time, these accoustic oscillations would produce

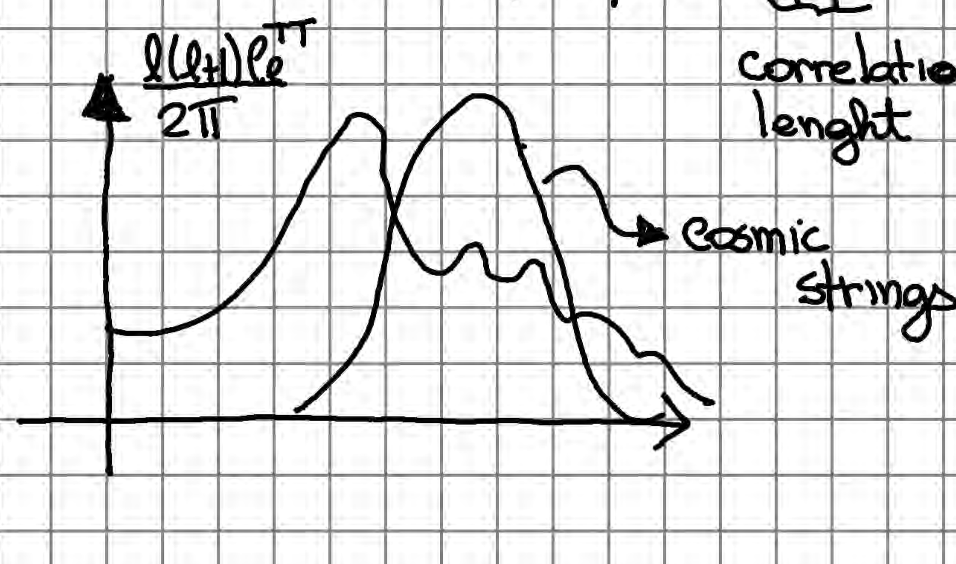
$$\frac{\Delta T}{T}(K,\eta_*) \sim \int_0^{\eta_*} d\eta_s\, A(K,\eta_s) \cos\left[K\left(r_s(\eta_*) - r_s(\eta_s)\right) + \phi(K,\eta_s)\right]$$

and then, how do we $\left\langle e^{i(\phi(K,\eta_1)-\phi(K,\eta_2))}\right\rangle \longrightarrow 0$

avoid decorrelation

when $(\eta_1-\eta_2) \gg \underbrace{\eta_T}_{\text{correlation lenght}}$

Example Cosmic strings $\downarrow$ just a blob



Here, don't distort my words: inflation predicts $\mathcal{R}(k)$ primordial curvature: $\mathcal{R}(k) = |\mathcal{R}(k)| e^{i\alpha_k}$

and α_k is random. ($\alpha_{k_1} \neq \alpha_{k_2}$)

$\hookrightarrow$ coherence means the oscillator has a deterministic phase relative to initial conditions

Imagine harmonic oscillator $X_k'' + k^2 c_s^2 X_k = 0$ $\left(X_k \sim \frac{\Delta T}{T} + \Psi \right)$

Solution $X_k = A_k \cos(k r_s) + B_k \sin(k r_s)$ or

$$\underline{X_k = C_k \cos(k r_s + \varphi_k)}$$

We would have been in deep trouble if φ_k was itself a random variable $\rightarrow$ on passive evolution it is <u>not</u> $\rightarrow$ φ_k is determine by cosmology via the Boltzman evolution

$\hookrightarrow$ All modes have the same phase origin that comes from a single stochastic variable $\mathcal{R}(k)$ which obeys $\langle \mathcal{R}(\vec{k}_1) \mathcal{R}^*(\vec{k}_2) \rangle = (2\pi)^3 \delta_D(\vec{k}_1 - \vec{k}_2) P_{\mathcal{R}}(k)$
(NG would be ok ☻)

One random variable means that

$$\delta_\gamma(k,\eta) = T_\delta(k,\eta)\mathcal{R}_k$$

$$v_\gamma(k,\eta) = T_{v_\gamma}(k,\eta)\mathcal{R}_k$$

$$\Phi(k,\eta) = T_\Phi(k,\eta)\mathcal{R}_k$$

$$\vdots$$

One random variable $\mathcal{R}_k$ for all physical quantities

One random number per k-mode. Everything else (transfer function) is deterministic

→ one starting time → one stochastic "kick"

→ The deterministic transfer can produce a smooth and deterministic phase shift (that is fine)

So in terms of oscillators we can think of all k's having same starting points & evolving under different speeds: so at recombination some k's will be at maximum compression, others at minimum (change with k is smooth)

How CMB measures H_0 to 1.5% precision

Let's come back to

$$\frac{\ell(\ell+1)C_\ell^{TT}}{2\pi} \sim \left\langle \frac{\Delta T}{T}(\hat{n}) \right\rangle^2 \sim A(\ell)\cos^2(\ell\theta_*)$$

θ_* we measure extremelly well $\sigma(\ln\theta_*) \sim \text{few} \times 10^{-4}$

$\theta_* = r_s/D(z_*)$, r_s is the comoving sound horizon

In the absence of new physics (e.g., Early Dark energy)

$$\frac{r_s}{\text{Mpc}} \approx 144.4 \left(\frac{\Omega_m h^2}{0.14}\right)^{-0.252} \left(\frac{\Omega_b h^2}{0.024}\right)^{-0.083}$$

We will see later that the distortion $A(\ell)$ of the $\cos^2(\ell\theta_*)$ give us

$100\,\Omega_m h^2 \sim 14.31 \pm 0.18$ (1.2% measurement)
$100\,\Omega_b h^2 \sim 2.237 \pm 0.015$

If you look at the exponents $\boxed{\sigma(\ln r_s) \approx \sigma(\ln \Omega_m h^2/4)}$

And because θ_* is so well measured that implies $\boxed{\sigma(\ln D(z_*)) \approx -\sigma(\ln \Omega_m h^2/4)}$

In flat LCDM, we know how to compute $D(z_*)$

↳ we have

$$\begin{cases} \text{Radiation}: \Omega_R h^2 (1+z)^4 \\ \text{Non-relativistic matter}: \Omega_m h^2 (1+z)^3 \\ \Lambda: (h^2 - \Omega_m h^2 - \Omega_R h^2) \quad \text{flatness closure} \end{cases}$$

↳ $h \equiv H_0/100$

and the CMB monopole pins $\Omega_R h^2$

$$\underbrace{D(z_*)}_{D_*} = \int_0^{z_*} dz' \Big/ \sqrt{\text{Rad} + \text{NR matter} + \text{closure}}$$

and if you do Taylor expansion around a fiducial cosmology, we get

$$\Delta \ln D_* \sim -0.20 \Delta \ln h - 0.40 \Delta \ln \Omega_m h^2$$

We saw from θ_* that $\Delta \ln D_* \sim -\Delta \ln \Omega_m h^2/4$

$$\boxed{(\Delta \ln h) \approx -0.71 \Delta \ln(\Omega_m h^2)}$$

$$\boxed{\sigma(\ln h) \approx 0.71 \cdot \sigma(\ln(\Omega_m h^2))}$$

→ $\Omega_m h^2 \sim 1.8\%$

Without additional physics to get better measurement of H_0, we need better measurement of $\Omega_m h^2$. the error bar relation is almost $1 \leftrightarrow 1$

The same calculation can also be written as

$\Delta\ln\theta_* = \Delta\ln r_s - \Delta\ln D(z_*)$ $\therefore$

$$\Delta\ln\theta_* \approx -0.252\Delta\ln(\Omega_m h^2) + 0.19\Delta\ln h + 0.40\Delta\ln(\Omega_m h^2)$$

$$\approx 0.20\Delta\ln h + 0.148\Delta\ln(\Omega_m h^2)$$

$$\approx \frac{1}{0.148}\left(1.35\cdot\Delta\ln h + \Delta\ln(\Omega_m h^2)\right)$$

So $\underline{\Delta\ln\theta_* \approx 0} \Rightarrow \boxed{\Omega_m h^{3.4} \approx \text{const}}$

$$\propto \underline{(\Omega_m h^2)\times h^{1.4}} \approx \text{const}$$

↳ That is why we need $\underline{\Omega_m h^2}$ from peak position and also lensing ($\Delta\ln C_L^{\phi\phi} \sim 2.4\Delta\ln\Omega_c h^2$). This is why optical depth of reionization became a hot topic on DESI-CMB. τ going up → A_s goes up ($A_s e^{-2\tau}$ = const on CMB) and $\Delta\ln C^{\phi\phi}$ goes up → $(\Omega_c h^2)$ $\underline{\text{down}}$

- if τ increases from $\tau \sim 0.054$ to 0.09 then $\Delta\ln A_s \sim 0.072$ and that means $\Omega_c h^2$ decreases by $\sim 2.4\%$ ($\Omega_c h^2 \sim 0.12$) → that is a 2σ shift

The central value is influence both by $\Omega_b h^2$ and $(\Omega_m h^2)$. but it is much more sensitive to bias in $(\Omega_m h^2)$ because of the -0.08 $\Omega_b h^2$ exponent in the sound horizon

Again later we will see how the distortion $A(\ell)$ of the $\cos^2$ get us $(\Omega_b h^2)$ and $(\Omega_m h^2)$

How BAO measures H_0

up to recombination baryon, photons were coupled

That means $\delta_b \sim \cos(k r_d)$. After "recombination" baryons felt into the gravitational potential of Dark matter, but they didn' go very far (they are non-relativistic)

Let's assume for now that baryons were frozen at ~recombination (so we use r_d)

$$\delta_b(k,\eta) \sim \cos(k r_d)$$

Now, let's take the 1D Fourier transform

$$\delta(x,\eta) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{ikx} \underbrace{\cos(k r_d)}_{\left(e^{ikr_d} + e^{-ikr_d}\right)/2}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \frac{e^{ik[(x+r_d)+(x-r_d)]}}{2}$$

Given that $\int_{-\infty}^{\infty} d\xi \, e^{i\xi} = 2\pi \delta_D(\xi)$; δ_D = Dirac delta

we get $\delta_D(x) = \frac{1}{2}\left[\delta_D(x-r_d) + \delta_D(x+r_d)\right]$

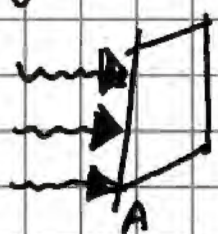
NR matter is also made of Dark matter, which just cluster in the center

$$\delta_m(x) \sim (1-f_b)\,\delta_D(x) + \frac{f_b}{2}\left[\delta_D(x-r_d) + \delta_D(x+r_d)\right]$$

f_b = baryon to total NR matter

$r_d \sim 1.018\, r_s$ Again $r_d > r_s$ by 2%

Why the 2% (technicality)



Area A

In a time t, $dE = u\,\overbrace{Acdt}^{\text{Volume}}$ (u = energy density)

$$\Rightarrow \underbrace{S = \frac{dE}{Adt}}_{\text{energy flux}} = cu$$

Radiation: $P = E/c$

$$\boxed{\text{So momentum flux} = \frac{dP}{Adt} = \frac{S}{c} = (\text{energy flux})/c}$$

Let $\Phi_\gamma = n_\gamma$/Area/time. Electrons have cross section σ_T (Thompson) $\Rightarrow$ scattering rate $\Gamma_\gamma = \Phi_\gamma \sigma_T$

- If the photon transfer average momentum Δp

the $\underbrace{F_e}_{\text{force one}^-} = \frac{dp_e}{dt} = \Gamma_{scat} \Delta p = \Phi_\gamma \sigma_T \Delta p$

- $\Phi_\gamma \Delta p$ is the momentum flux $= \vec{S}/c$

$\Rightarrow$ $\boxed{F_e = \sigma_T \frac{\vec{S}}{c}}$, $\vec{S}$ = energy flux

↳ In a perfect collimated beam $\vec{S} = uc$ as each γ travels at the speed of light.

↳ In an perfect isotropic field $\vec{S} = 0$

↳ With some dipole anisotropy in the radiation fied, the bulk velocity is V_γ which maps into the momentum transfer is $\ll c$

↳ how it maps? $S = (u_\gamma + P_\gamma) V_\gamma$ if the electron is at rest and near isotropic radiation (w/ dipole) is moving slowly.

- In e^- frame

$V_\gamma^{(e)} = V_\gamma - V_e$

$$\text{So } F_e = \frac{\sigma_T}{c}(u_\gamma + p_\gamma)(V_\gamma - V_e) \overset{(p_\gamma = u_\gamma/3)}{=} \frac{4\sigma_T}{3c}(V_\gamma - V_e)u_\gamma$$

$$\boxed{F_e = \frac{4\sigma_T}{3c}(V_\gamma - V_e)u_\gamma}$$ force in a single electron

Let n_e be the density of electrons

Force per unit of volume
$$f_{\gamma e} = n_e F_e = \frac{4}{3} n_e \sigma_T u_\gamma \left(\frac{V_\gamma - V_e}{c}\right)$$

- $V_e \sim V_b$ (Coulomb Forces would prevent this deviation)

$$\left.\frac{dV_b}{dt}\right|_{drag} = \frac{\frac{4}{3} n_e \sigma_T u_\gamma \left(\frac{V_\gamma - V_b}{c}\right)}{\rho_b}$$ and $\underbrace{\rho_b}_{\text{mass density}} c^2 = \underbrace{u_b}_{\text{energy density}}$

$$= \underbrace{\left(\frac{4}{3}\frac{u_\gamma}{u_b}\right)}_{\equiv 1/R} n_e \sigma_T c (V_\gamma - V_b)$$

Now $n_e c \sigma_T \sim$ scattering rate so

$$\boxed{\Gamma_{drag} \sim \Gamma_\gamma / R} \rightarrow \boxed{d\tau_d = d\tau_\gamma / R}$$

τ_γ optical depth of radiation (physical meaning:

τ_γ attenuates intensity

$$I(t_2) = I(t_1)\, e^{-\Delta t/\tau_\gamma} \quad (\tau_\gamma \text{ physical meaning})$$

- $d\tau_d$: attenuates relative bulk velocity

$$\Delta v \equiv v_b - v_\gamma \qquad \Delta v(t_2) = \Delta v(t_1)\, e^{-\Delta t/\tau_d}$$

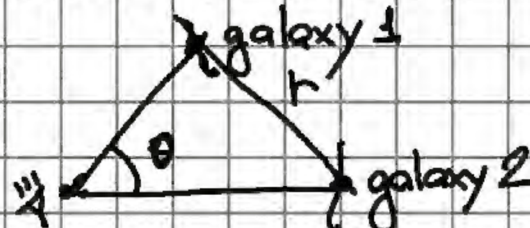
Remember (after this appendix on r_d)

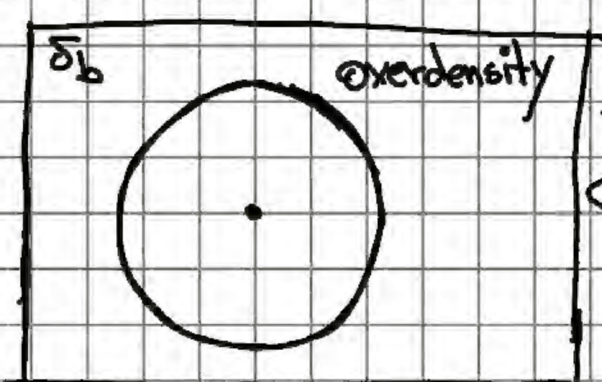
$$\delta_b(x) = \underbrace{(1-f_b)\,\delta_D(x)}_{\substack{\text{central}\\ \text{peak (dark matter}\\ \text{cluster in the}\\ \text{center attracting baryons)}}} + \frac{f_b}{2}\underbrace{\left[\delta_D(x-r_d) + \delta_D(x+r_d)\right]}_{\substack{\text{bump (spherical}\\ \text{bump in 3D)}}}$$

↳ This is around every point in space

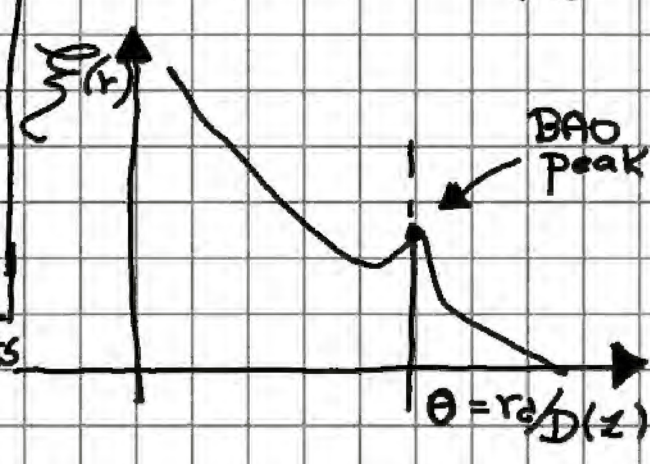
Then we can compute correlation function

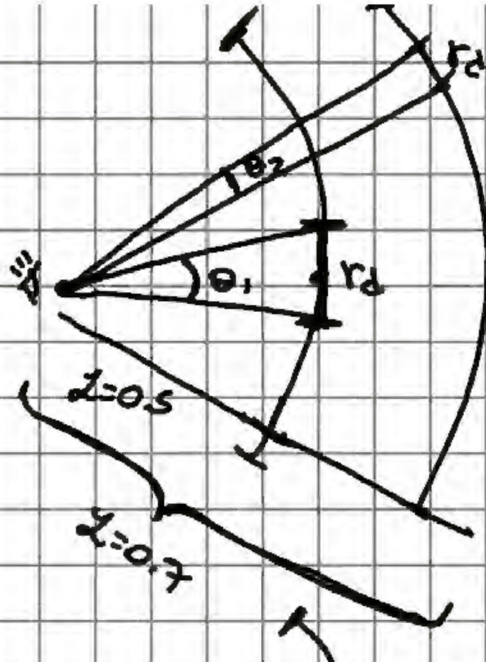
$$\xi(r) \equiv V^{-1}\int dx\, \delta_b(x)\,\delta_b(x+r)$$





Standard ruler in r_d units

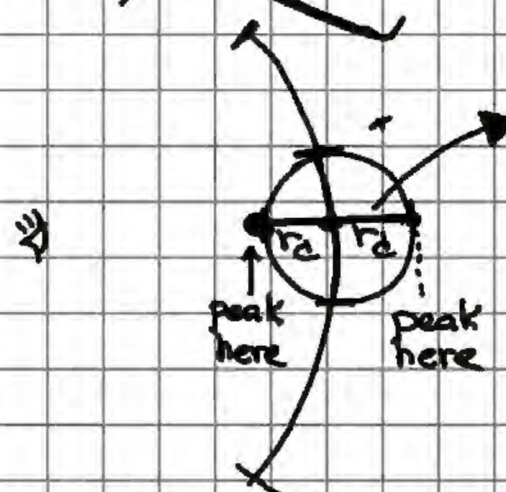




- This is transversal BAO
- There is also radial BAO

↳ $\theta = r_d / D_A$ (comoving angular diameter distance)

$D_A = D$ in flat universe



radial BAO

↳ measures $\delta H(z)$ on δz on which $\delta D \sim 2 r_d$

↳ $\delta z / 2 \sim r_d \Big/ \left.\frac{dD}{dz}\right|_{z_c}$ center

In this talk, I will focus on transversal BAO, which measures $\theta = r_d / D(z)$

For z slices at $z \ll 1$, $D(z) \sim c\bar{z}/H_0$ ($\bar{z}$ = slice z)

and $r_d \approx r_s = 144.4 \left(\frac{\Omega_m h^2}{0.14}\right)^{-0.252} \left(\frac{\Omega_b h^2}{0.024}\right)^{-0.083}$

$$\theta \propto \underbrace{\left(\frac{H_0}{100}\right)}_{h} r_d = \boxed{h r_d \propto (\Omega_m h^2)^{-0.25} (\Omega_b h^2)^{-0.08}}$$

So you need $\Omega_b h^2 \rightarrow$ get from BBN

Now you know how to estimate H_0 from

- SHOES (Adam Riess)
- CMB
- BAO

$\Bigg\}$ If you get the cosmology right, they should all agree

Why H_0 is Dark Energy

(Talk from WH 2006)

- Λ, $w=-1$, $\rho_\Lambda = \text{const}$
- $w_0 w_a$ (CPL), $w(z) = w_0 + \dfrac{w_a z}{1+z}$, $\dfrac{\Omega_\Lambda}{\Omega_\Lambda^0} = e^{3\int_0^z d\ln z\,(1+w(z))}$

$$\boxed{w(z\to 0)\to w_0,\ w(z\gg 1)\to w_0+w_a}$$

$$\frac{\Omega_{DE}(z)}{\Omega_{DE}^0} = (1+z)^{3(1+w_0+w_a)}\, e^{\left[-3w_a z/(1+z)\right]}$$

We can see that $z\gg 1$, $\left[e^{-3w_a z/(1+z)}\right] \to \text{const}$

- $\boxed{\Omega_{DE}(z\gg 1) \propto (1+z)^{3(1+w_0+w_a)} \Big\}\ \text{const}\ (w_0+w_a)\ \text{EOS}}$

$$\frac{\Omega_{DE}(z)}{\Omega_{DE}^{\circ}} = (1+z)^{3(1+w_0+w_a)}\, e^{\left(-3w_a z/(1+z)\right)}$$

$$(1+z)^{3(1+w_0+w_a)} \xrightarrow[z\to 0]{} 1 + 3(1+w_0+w_a)z$$

$$e^{\left(-3w_a z/(1+z)\right)} \xrightarrow[z\to 0]{} 1 - 3w_a z$$

$$\boxed{\frac{\Omega_{DE}(z)}{\Omega_{DE}^{\circ}} \xrightarrow[z\to 0]{} 1 + 3(1+w_0)z \quad \text{(const } w_0 \text{ solution)}}$$

Well, it turns out that is not the best way to understand CPL. Why? Because we need to consider that $D(z_*)$ is fixed (ie, we will always add CMB)

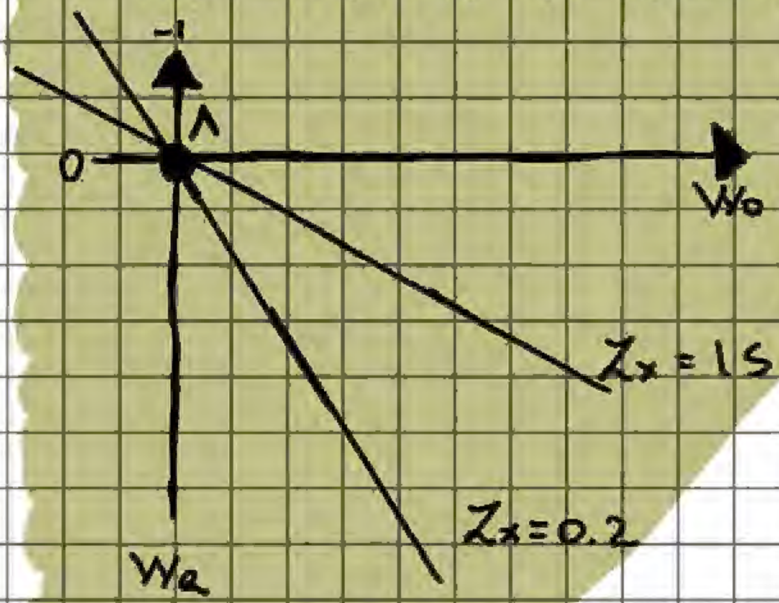
$D(z_*) = $ const. To first order around $w_0=-1, w_a=0$

$$\delta D \approx \frac{\partial D}{\partial w_0}(w_0+1) + \frac{\partial D}{\partial w_a} w_a = 0$$

$$\rightarrow \boxed{w_a = -A(1+w_0)} \qquad A = \left.\partial D/\partial w_a \Big/ \partial D/\partial w_0\right|_{z=z_*}$$

We will consider it a free parameter

$w_a = -A(1+w_0)$ to maintain $D(z_*)$ fixed



What is z_x?

That is the so-called phantom crossing

Now let's compute z_x at which $W(z) = -1$

$$W(z) = w_0 + \frac{w_a z}{1+z}; \quad \underbrace{-1 - w_0}_{-(1+w_0)} = \frac{w_a z_x}{1+z_x};$$

Now $w_a = -A(1+w_0)$; therefore

$$1 = \frac{A z_x}{(1+z_x)} \rightarrow (1+z_x) = A z_x \rightarrow 1 = z_x (A-1)$$

So A is directly related to z_x; $\boxed{z_x = \frac{1}{(A-1)}}$

When you see DESI + CMB havin countours like that is in part just D_* being fixed.

Ok but why then H_0 is dark energy?

Well, with $w_a \approx -4(1+w_0)$ you compute $\frac{\Delta D(z)}{D(z)}$

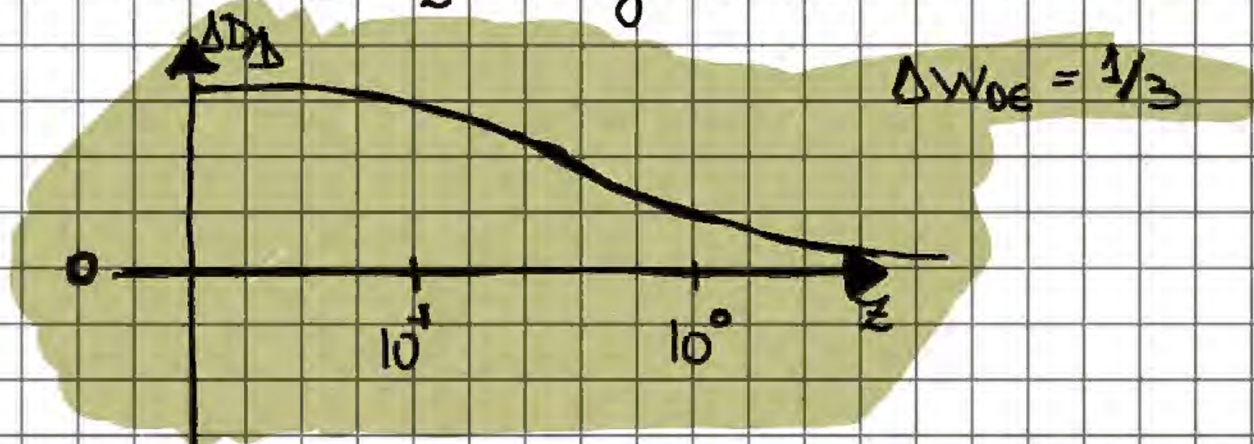
Compare to ΛCDM and you find that

except for $(1+w_0) \sim -0.27\, w_a$, we predict

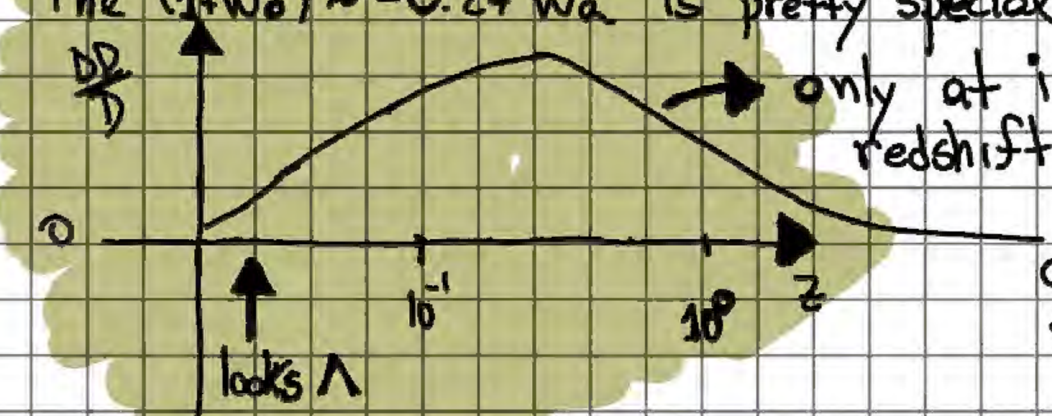
H_0^{CMB} to be different than measure by Riess

Furthermore, if $w(z) =$ const, we can ask at which

redshift $\Delta D/D|_z$ is largest. Answer: $z=0$.



The $(1+w_0) \sim -0.27\, w_a$ is pretty special (mirage)



→ only at intermediate redshift you see a difference compared to LCDM for other values of w_0, w_a around $(-1, 0)$

$$\Delta \ln H_0 \sim -0.447(1+w_0) - 0.121\,\underbrace{w_a}_{-A(1+w_0)}$$

$$\Delta \ln H_0 \sim (1+w_0)\left(-0.447 + 0.121\,\underbrace{A}_{(1+z_x)/z_x}\right)$$

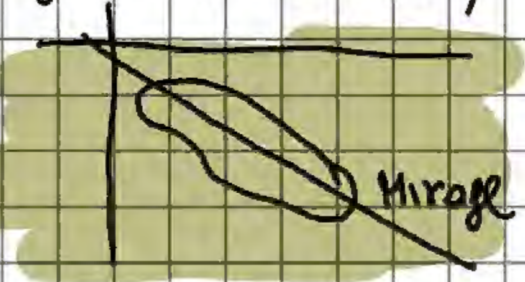
$$\Delta \ln H_0 \sim -0.447(1+w_0)(1-0.27A), \quad \boxed{A \approx 3.70}$$

$$\boxed{\Delta \ln H_0 \sim -0.447(1+w_0)\left(1-0.27\,\frac{(1+z_x)}{z_x}\right)}$$

For Mirage $\boxed{z_x = \frac{1}{A-1} \approx \frac{1}{1/0.27-1} \approx \frac{1}{2.70} \simeq 0.37}$

If low-z experiments, when assuming $w = \text{const}$ get $w \approx -1$ (like DESI $w \approx -0.997 \pm 0.027$) that means z_x is low (like ~ 0.5) and that means preferred models is close to mirage. that is exactly what we see w/ DESI

$\boxed{\text{Linder 0708:0024}}$



Why Dark energy fails to solve the H_0 tension

Riess $H_0 \approx 73$ Km/s/Mpc, BAO $H_0 \approx 68$ Km/S/Mpc
($z_{BAO} \sim 0.5$)

• Key: Supernove. the Phillips relation allow us to recalibrate SN relative differences in brightness (simplify: you can think we can select a subsample of SN w/ similar Δm_{15} so they have the same relative brightness)

→ in this subsample from $z \approx 0.01$ to $z \approx 0.5$ we can predict in a LCDM universe w/ $H_0 \approx 73$ what should be the z dependency of their apparent magnitudes (apparent brightness)

Now if DE "glues" 2 LCDM models one w/ $H_0 = 73$ Km/s/Mpc at $z \approx 0.01$ to another w/ $H_0 \approx 67$ Km/s/Mpc we should have seen the apparent magnitudes drifting from expectations by $\approx m_{67} - m_{73} = 5\log_{10}\left(\frac{73}{67}\right) \approx 0.183$

change in apparent magnitude

a Δm drift of 0.123 is enormous compared to current error bars !!!

Why Early Dark Energy works?

At zero order explanation is simple. We have

$r_d \simeq 1.02\, r_s$ and $r_s = \int_{z_*}^{\infty} \frac{dz' \, c_s(z')}{H(z')}$. So if we

introduce an Early Dark energy that changes $H(z)$ we can shift it by $\frac{\Delta H_0}{H_0} \sim 8\%$ and we are done.

→ define $f_{EDE}(a) \equiv \rho_{EDE}(a) / \rho_{total}(a)$

→ $\rho_T = \rho_{std} / (1 - f_{EDE})$

Given that $H^2 \propto \rho \Rightarrow H^2_{EDE} = H^2_{std} / (1 - f_{EDE})$

→ $\frac{1}{H_{EDE}} = \frac{\sqrt{1 - f_{EDE}}}{H_{std}} \sim \frac{(1 - f/2 - f^2/8)}{H_{std}}$

Define normalize sound Horizon as

$$W(a) \equiv \frac{1}{r_s^{std}} \frac{c_s(a)}{a^2 H(a)} \Rightarrow \int_{z_*}^{\infty} dz\, W(z) = 1$$

$$\text{so } \frac{r_s^{EDE}}{r_s^{std}} \sim \underbrace{\int_{z_*}^{\infty} dz\, W(z) \sqrt{1 - f_{EDE}}}$$

$$\approx 1 - \frac{1}{2} \int_{z_*}^{\infty} dz\, W(z) f_{EDE} \approx 1 - \frac{\langle f_{EDE} \rangle}{2}$$

$$\boxed{\frac{\Delta r^{EDE}}{r_s^{std}} \sim -\frac{1}{2} \langle f_{EDE} \rangle} \rightarrow \text{fix } \theta_* \quad \frac{\Delta H_0}{H_0} \sim \frac{1}{2} \langle f_{EDE} \rangle$$

$\boxed{\text{So at fix cosmo, 10\% } f_{EDE} \text{ raises } H_0 \text{ by 5\%}}$

However EDE changes $\Omega_m h^2$ in a way it is relevant for DESI

↳ How? EDE does 3 things: 2 of them are larger driving force & ISW

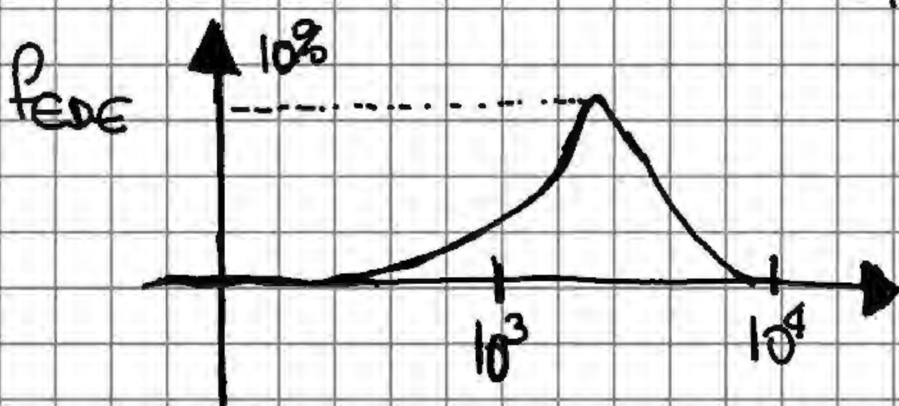
What is EDE (Early Dark Energy)

- EOS in its simplest form

$$w_{EDE}(a) = \frac{1 + w_f}{1 + \left(\frac{1+z}{1+z_c}\right)^{3(1+w_f)}} - 1$$

$z \gg z_c \quad w_{EDE} \to 1 + w_f \qquad z_c \sim z_{MR} \sim 3800$ matter radiation equality

$z \ll z_c \quad w_{EDE} \to -1$



Remember $H_{EDE}(z) = H_{std}(z) / \sqrt{1 - f_{EDE}(z)}$

$$\Rightarrow \boxed{r_s^{EDE} \sim \left(1 - \frac{\langle f^{EDE} \rangle}{2}\right) r_s^{std}} \quad \text{as } r_s \propto \int 1/H$$

However, EDE affects the CMB in deeper ways (let's dive in)

↳ Accoustic Physics in 20 min

$\Theta(k,\eta) \equiv \frac{\Delta I}{T}(k,\eta) = \frac{1}{4}\delta_\gamma(k,\eta)$ is a forced accoustic oscillator

(η is conformal time)

Hu-Sugiyama Equation (η = conformal time, $R_b = \frac{3}{4}\frac{\rho_b}{\rho_\gamma}$)

$$\underbrace{\left[\frac{d^2}{d\eta^2} + \left(\frac{R'}{1+R}\right)\frac{d}{d\eta}\right.}_{c_s^2 \frac{d}{d\eta}\left[c_s^{-2}\frac{d}{d\eta}\right]} \left. + \underbrace{k^2 c_s^2}_{\omega^2}\right] \Theta = -\frac{k^2 c^2 \Psi}{3} - c_s^2 \frac{d}{d\eta}\left(c_s^{-2}\dot{\Phi}\right)$$

Φ, Ψ the potentials in Newtonian Gauge

& $c_s = (c/3)/\sqrt{1+R}$ $(c=1)$ $\quad \delta g_{00} = -2a^2\Psi, \delta g_{ii}\ 2a^2\Phi$

→ If this look unfamiliar: start w/ Lagrangian

$\mathcal{L} = \frac{1}{2}m(t)\dot{x}^2 - \frac{1}{2}k_{spring}x^2$ with $k_{spring} = m\omega^2 \xrightarrow{HO} \frac{1}{m}\frac{d}{dt}(m\dot{x}) + \omega^2 = 0$

WKB George mentioned in the 1st week: possible b/c there are 2 scales

Fast vs Slow solution

Φ^f, Θ^f	Φ^s, Θ^s	Example $R \propto \rho_b/\rho_\gamma$
Vary on ω timescale (deep inside horizon)	Vary on $\mathcal{H}$ timescale	$R \propto a \Rightarrow \frac{d\ln R}{d\eta} = \mathcal{H}$
$\frac{dX^f}{d\eta} \sim \omega X^f$	$\frac{dX^s}{d\eta} \sim \mathcal{H} X^s$	

On $\Psi = \Psi^s + \Psi^f$: Which components vary on $\mathcal{H}$ and ω timescales? {Slow: dark matter! Fast: baryon-γ fluid

→ So Ψ^s is sourced by DM.

→ Ψ^f? $k^2\Psi^f = -4\pi G a^2 (\rho_b\delta_b + \underbrace{\rho_\gamma\delta_\gamma}_{\sim X^2})$. Adiabatic perturbation $\Rightarrow \delta_b = \frac{3}{4}\delta_\gamma \rightarrow k^2\Psi^f = -4\pi G a^2 \rho_\gamma \delta_\gamma (1+R)$

For WKB: fast mode obeys the homogeneous equation
Why? $\delta_\gamma \sim O(1) * \cos(\varphi(\eta)) \Rightarrow \delta_\gamma$ does not grow!
So Poisson tell us that $\Psi^f \sim (\mathcal{H}/k)^2 \delta_\gamma \rightarrow 0$ on WKB

Ansatz for homogeneous equation:

$$\Theta_f(k,\eta) = A(\eta) e^{\pm i\varphi(\eta)}; \quad \text{with } \varphi(\eta) = k\int_0^\eta d\eta' c_s(\eta')$$

- In the slow solution we neglect derivatives of Ψ, Φ

$$\underbrace{\Theta_0^{s\,''} + \frac{R'}{(1+R)}\Theta_0^{s\,'}}_{c_s^2(c_s^{-2}\Theta_0^s)'} + k^2 c_s^2 \Theta_0^s \approx -\frac{k^2 c^2}{3}\Psi_s \quad \text{w/ } c_s^2 \propto 1/m$$

Dropping all derivatives of slow modes ($\sim$ const. on $1/\omega$ timescale)

$$k^2 c_s^2 \Theta_0^s \approx -\frac{k^2 c^2}{3}\Psi_s \Rightarrow \Theta_0^s = -\frac{c^2}{3c_s^2}\Psi_s = -(1+R)\Psi_s$$

Therefore the WKB solution is

$$\boxed{\Theta_\gamma(k,\eta) = (1+R)^{-1/4} C \cos\varphi(\eta) - (1+R)\Psi}$$

WKB (valid when modes are deep inside the Horizon)

$\hookrightarrow$ Even when $R=0$, WKB still valid (where Ψ^s is not const but decays as $\Psi^s \sim \ln a/a$). Point is $\Psi^s \sim$ const on $1/\omega$ time scale. WKB sets the tone: baryons change ω and amplitude / Ψ shifts the oscillator. But hard to see here

$\dot{\Psi}, \dot{\Phi}$ around horizon crossing (where $\dot{\Psi} \sim \mathcal{H}^{-1} \sim \omega$), where WKB is not valid. EDE is expected to influence this driving effect.

Initial condition is given by super horizon solution (GR). On modes that enter horizon during matter dominance ($B \& R_b = 0$)

$$\Psi = \frac{3}{5}\mathcal{R}, \quad \delta_\gamma = -\frac{8}{5}\mathcal{R} \Rightarrow \Theta = -\frac{2}{5}\mathcal{R}$$

Therefore: $\Theta \to C - \Psi = -\frac{2}{5}\mathcal{R} = C - \frac{3}{5}\mathcal{R}$

$$\Theta = (1+R_b)^{-1/4} \cos\left(k\int_0^{\eta} c_s(\eta)\,d\eta\right) - (1+R)\Psi$$

$A = (1+R_b)^{-1/4}$: amplitude of oscillation decays adiabatically as R increase. Another way to see that is that in classical mechanics $\frac{E}{\omega}$ is an adiabatic invariant

$E = \frac{1}{2}\frac{\overbrace{m\omega^2}^{K_{spring}} x^2}{\omega}$, mass of H.O.; $m \propto c_s^{-2} \propto (1+R_b)$ $\Rightarrow A^2\sqrt{1+R_b} = \text{const}$

$\omega \propto c_s \propto 1/\sqrt{1+R_b}$ $\quad A \propto 1/(1+R)^{1/4}$

$x^2 = A^2$ (amplitude)

$\hookrightarrow c_s \propto (1+R_b)^{-1/2} \Rightarrow \theta_* \propto (1+R_b)^{-1/2}$ ($R_b \approx \text{const}$) $\Rightarrow l_A \propto (1+R_b)^{1/2}$ (shift r_s!)

Hu & Sugiyama solution (Green Function from Wronskian)

$$(1+R)^{1/4}\Theta(\eta) = \Theta(0)\cos(k r_s)$$
$$+\frac{\sqrt{3}}{kc}\int_0^{\eta} d\eta' \,(1+R_b(\eta'))^{3/4} \ast \sin[k(r_s(\eta)-r_s(\eta'))]\,F(\eta')$$

with $\boxed{F(\eta) = -\ddot{\Phi} - \frac{\dot{R}}{1+R}\dot{\Phi} - \frac{k^2c^2}{3}\Psi}$ $\quad R_b = \frac{3}{4}\rho_b/\rho_R$

To practice, make a bit wrong but simple approximation: R_b = const (start w/ R_b=1). This is wrong (gets factor a bit wrong) but fun & good practice

↳ Wayne calls this the Adiabatic Approximation

Limiting solutions: (toy solution) $\boxed{R_b = 1} \Rightarrow c_s = \frac{c}{\sqrt{6}}$, $r_s = c_s\int_0^{z_*}\frac{dz'}{H(z')} = c_s\eta_*$

so $k r_s = \overbrace{k c_s}^{\omega}\eta \equiv \omega\eta$, $F = -\frac{k^2c^2}{3}\Psi \quad (\dot{\Phi} = \ddot{\Phi} = 0)$ ∴

$$\Theta(\eta) = \Theta(0)\cos(\omega\eta) - \frac{kc}{3}\Psi\sqrt{6}\int_0^{\eta} d\eta' \sin[\omega(\eta'-\eta)]$$

$$= \Theta(0)\cos(\omega\eta) - \frac{(kc)^2}{3}\frac{\Psi}{\omega}\int_0^{\eta} d\eta' \sin[\omega(\eta'-\eta)]$$

- $u \equiv \omega(\eta'-\eta)$ the integral is $\frac{1}{\omega}\underbrace{\int_0^{x}\sin u\, du}_{-\cos(u)\big|_0^{x} = (1-\cos x)}$

So $\Theta(\eta) = \Theta(0)\cos(x) - \overbrace{\frac{k^2c^2}{3\omega^2}}^{2}\Psi\,(1-\cos(x))$

$$\Theta(\eta) = (\Theta(0) + 2\Psi)\cos(x) - 2\Psi\;;\; x = kc\eta/\sqrt{6}$$

IC: given by superhorizon adiabatic modes

$$\begin{cases} \mathcal{R} = \dfrac{(5+3w)}{(3+w)}\Psi_i \overset{w=0}{\Rightarrow} \Psi_i = 3\mathcal{R}/5 \text{ and } \Psi_i = \Psi \text{ as } \dot\Psi = 0 \\ \Theta_i = -\dfrac{2}{5}\mathcal{R} \end{cases}$$

R_b const : adiabatic approx

easily extended to any R_b const

Final Solution:

$$\boxed{(\Theta + \Psi)(x) = \frac{4}{5}\mathcal{R}\cos x - \frac{3}{5}\mathcal{R}}$$

for an R = const solution

$$\begin{cases} \frac{4}{5}\mathcal{R} \rightarrow \frac{(3R_b+1)}{5}\mathcal{R} \\ 3/5 \rightarrow 3R_b/5\,\mathcal{R} \end{cases}$$

A bit wrong compare to WKB

- Another limiting solution ($R_b = 0$)

↳ Perturbation says Ψ, Φ decays inside the Horizon.

↳ Define $f(x) = 3\left(\dfrac{\sin x - x\cos x}{x^3}\right)$. Neglecting neutrinos $\Psi = -\Phi$

and $\Psi(x) = \Psi_i f(x)$ with $\mathcal{R} = \underbrace{\dfrac{5+3w}{3(1+w)}}_{\sim 6/4 = 3/2}\Psi_i$ w/ $w = 1/3$

ie, $\Psi_i = \dfrac{2}{3}\mathcal{R}$

Also superhorizon in Radiation era $\boxed{\Theta_i = -\frac{\Psi_i}{2}}$

So the solution is

$$\frac{\Theta}{\Psi} = -\frac{1}{2}\cos x + \int_0^x du \sin(x-u)\left[\overbrace{f_{,uu}(u)}^{d^2f/du^2} - f(u)\right]$$

Complicated expression

$$\frac{\Theta}{\Psi_i} = \frac{3}{2}\cos x - 3\frac{\sin x}{x} - \frac{3\cos(x)}{x^2} + \frac{3\sin x}{x^3}$$

- $x \to 0 \quad \frac{\Theta}{\Psi_i} \to \frac{3}{2} - 3 - \frac{3}{x^2}\left(1 - \frac{x^2}{2}\right) + \frac{3}{x^3}\left(x - \frac{x^3}{6}\right)$

$$\frac{\Theta}{\Psi_i} \to -\frac{3}{2} + \underbrace{\frac{1}{x^2}\left(-3+3\right)}_{0} + \frac{3}{2} - \frac{1}{2}$$

$\boxed{\frac{\Theta}{\Psi_i} \xrightarrow[x\to 0]{} -\frac{1}{2}}$ (consistent w/ superhorizon)

- $x \to \infty,\ \frac{\Theta}{\Psi_i} \to \frac{3}{2}\cos x \Rightarrow \Theta \xrightarrow[x\to\infty]{} \tilde{\mathcal{R}}\cos x$ comoving potential

The only limit we have not explored is $K \ll K_{eq}$ and $R = 0$ (relevant because $\delta_b \ll \delta_m$). We can set Ψ, Φ const and $R = 0$, and we get $\Theta(x) + \Psi = \frac{\mathcal{R}}{5}\cos x$

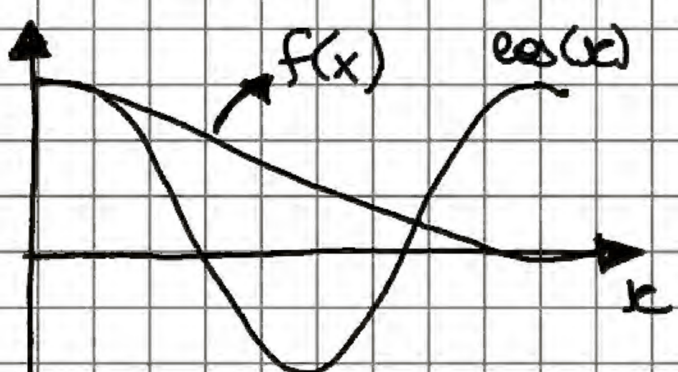
So the 3 solutions we have

- $K \ll K_{eq}$: $\Theta + \Psi = \begin{cases} \frac{\mathcal{R}}{5}\cos x & \text{for } R_b = 0 \\ \left(\frac{3R_b+1}{5}\right)\mathcal{R}\cos x - \frac{3R_b}{5}\mathcal{R} & \text{for } R_b = \text{cons} \end{cases}$

Most imp: $R_b = 0$ $(\Theta + \Psi) \to \frac{1}{5}\mathcal{R} \times \cos x$

- $K > K_{eq}$, $x \gg 1$: $\Theta(x) \approx (\Theta + \Psi) \to \mathcal{R}\cos x$

There is a boost ~ 5. Why? Decay of Ψ is in phase. Below, we plot $\cos x$ against $f(x)$



Another amazing fact from coherent phases

WKB does not see that. On WKB, $F(\eta) \approx$ const on one oscillation. So we only have integral of the Green Function (a sine) which is ≈ 0 over 1-oscillation

This was complicated: Is that a simpler way to see that? yes! Adiabatic approx. $R_b \approx$ const

$$\left(\frac{d^2}{d\eta^2} + k^2 c_s^2\right)(\Theta + \Psi) = (\Psi'' - \Phi'') \equiv F'' \Longrightarrow$$

$$(\Theta+\Psi)(k,\eta) = (\Theta+\Psi)(0)\cos(\omega\eta) + \frac{1}{\omega}\int_0^{\eta} F'' \sin[\omega \overbrace{\Delta\eta}^{\eta-\eta'}]$$

Expand: $\sin[\omega(\eta-\eta')] = \sin[\omega\eta]\cos(\omega\eta') - \cos(\omega\eta)\sin(\omega\eta')$

$\therefore (\Theta+\Psi)(k,\eta) = A_c \cos(\omega\eta) + A_s \sin(\omega\eta)$ with

$$A_c = (\Theta+\Psi)(0) - \frac{1}{\omega}\int_0^{\eta} F''\sin(\omega\eta')d\eta'$$

$$A_s = \underbrace{\frac{1}{\omega}\int_0^{\eta} F''\cos(\omega\eta')d\eta'}_{\frac{1}{\omega}[F'\sin] - \int d\eta' F'\sin(\omega\eta')} \quad \underbrace{+[F'\sin\omega\eta']\frac{1}{\omega}\Big|_0^{\eta}}_{\text{vanishes before \& after the decay}} + \int_0^{\eta} d\eta' F'\cos(\omega\eta')$$

Final Solution

$$\left.\begin{aligned} \bullet A_c &= (\Theta+\Psi)(0) + \int_0^{\eta} d\eta' F'\cos(\omega\eta') \\ A_s &= -\int_0^{\eta} d\eta' F'\sin(\omega\eta') \end{aligned}\right\}$$

Fully coherent means $F' \neq 0$ when $\cos(\omega\eta') \sim 1$

$\Rightarrow A_s \approx 0$ & $A_c \approx (\Theta+\Psi)(0) + \Delta F$; $\Delta F = \Delta\Psi - \Delta\Phi$

$\therefore \Delta F$ is the constructive change of potentials. Max change? Max change is $-2\Psi_i$ & $\Theta_i = -\frac{1}{2}\Psi$

$$\Rightarrow |(\Theta_i + \Psi_i + \Delta F)| \xrightarrow[\text{maximum}]{} |\tfrac{1}{2}\Psi_i - 2\Psi_i| = \tfrac{3}{2}|\Psi_i| = R$$

The "last" effect on accostic oscillation is Silk damping (does not exist on instanteneous recombination)

Near z_* photons were not strongly coupled to baryons:

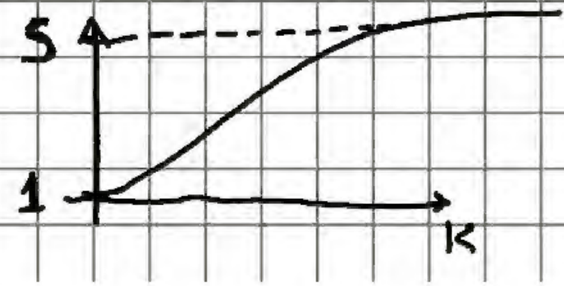
Comoving mean free path of γ : $\lambda_c = \frac{1}{n_e \sigma_T a} = 1/\dot{\tau}$ (conformal opacity)

Random Walk $\left\{ N \begin{pmatrix} \text{number} \\ \text{of collisions} \end{pmatrix} \sim \frac{\eta}{\lambda_c} \right.$ → Comoving horizon size → like D just different integration limits $\eta = \int_z^{\infty} dz/H(z)$

$$\lambda_D \sim \sqrt{N}\,\lambda_c \sim \sqrt{\eta \lambda_c} \sim 7.2\ \text{Mpc}$$

So you can think the accoustic oscillations as

$\underbrace{\Theta(k,\eta)}_{\text{The 3D } \frac{\Delta T}{T}(k,\eta)} \sim \cos \times$ Amplitude envelope that goes from 0 → 5 from large to small scals ↓ because potential decays $\times \underbrace{e^{\left(-(k/k_D)^2\right)}}_{k_D \sim \frac{2\pi}{\lambda_D}}$



CMB is a spherical projection.

This is the 3D $\frac{\Delta T}{T}(k,\eta)$ that gets frozen at recombination. There is no time to detail the calculation, but schematically:

$$\frac{\delta T}{T}(\hat{n}) = \frac{\delta T}{T}\Big|_{\text{frozen at recombination}} + \underbrace{\int_{\eta_*}^{\eta_0} d\eta\,(\dot{\Psi}-\dot{\Phi})}_{}$$

$\eta_0 \to$ today

Here you need to decompose as $\int dk\, e^{ik\hat{n}D_*} \frac{\delta T}{T}(k)$

If potential decay while photons travel they gain energy (ISW)

- So one must ask the question to EDE
  - Does EDE change $(\delta T/T)$ frozen at recombination
    - yes? How?
      - change Ψ, Φ? Or $\dot{\Psi}, \dot{\Phi}$?
      - change damping scale?
  - Does EDE change the propagation of photons?
    - ie, does EDE change ISW?

The answer is yes, which affect $\Omega_m h^2$ and n_s

So, EDE is not just reducing r_s

Indeed, we know from MCMC chains that EDE increases $\omega_m h^2$ and n_s. Let's try to understand it

↳ EDE changes damping to horizon ratio

$\theta_* = \frac{r_s}{D_*}$, and r_s shrinks as $r_s \to r_s/q$,

with $q = \frac{1}{\left(1 - \frac{\langle f_{EDE} \rangle}{2}\right)}$. Therefore, to maintain

θ_* fixed, H_0 increases as $H_0 \to H_0 q$

$\to \boxed{D \to D/q}$

OK Now Let's check $\theta_{damping} = r_{damping}/D_*$. In a random walk, r_{damp} decreases as $q^{-1/2}$ (ie squared root of conformal time $\to \sqrt{H^{-1}}$ as $\eta = \int dz/H$. Then,

$$\theta_d = \frac{r_d}{D_*} \to \frac{r_d/q^{1/2}}{D/q} \to q^{1/2}\theta_d$$

if $q \sim 1.1$, then $\sqrt{q} \sim 1.049 \Rightarrow \theta_d$ increases by 5%

If at fixed θ_s, θ_d increases by 5%, then l_D decreases by 5% → C_ℓ gets more damped ➡ n_s needs to increase to compensate it !

. Why $\Omega_m h^2$ changes: At m-R equality $\rho_m(z_{eq}) = \rho_R(z_{eq})$.

Without EDE then $\Omega_m(z_{eq}) = 1/2$. Now with EDE

$\Omega_m(z) \equiv \rho_m / \rho_{total}$ and $f_{EDE} \equiv \rho_{EDE}/\rho_{total}$, with

$\rho_{total} = (\rho_m + \rho_R) + \rho_{EDE}$ ➡ $\rho_{EDE} \frac{(1-f_{EDE})}{f_{EDE}} = \rho_m + \rho_R$

so: $\rho_{EDE} = \frac{f_{EDE}}{(1-f_{EDE})}(\rho_m + \rho_R)$

Now from definition, it is easy to see that:

$$\Omega_m(z) = \frac{\rho_m}{\rho_{total}} = \frac{\rho_m f_{EDE}}{\rho_{EDE}} = \frac{\rho_m}{(\rho_m + \rho_R)} \times (1 - f_{EDE}).$$

Ω_m reduced by $(1-f_{EDE})$ → stronger Ψ, Φ evolution and that implies stronger accoustic driving

EDE can still affect early ISW at recombination

↳ it actually does. Why?

↳ $\frac{a_{\text{recombination}}}{a_{\text{matter-recombination}}} = \frac{(1+z_{me})}{(1+z_*)} \sim 3.3$

with $w_{EDE} \sim \frac{1}{2}$ $\rho_{EDE} \propto (1+z)^{3(1+w_{EDE})} = (1+z)^{9/2}$

so $\frac{\rho_{EDE}}{\rho_m} = \left(\frac{1+z_*}{1+z_{mR}}\right)^{3/2} \sim 1/6 \sim 0.17$. So if $f_{EDE}^{max} \sim 10\%$, and $w_{EDE} \sim 1/2$, there is still 1-2% f_{EDE} at $\underset{\text{recombination}}{\tilde{z}_*}$

↳ There is also a memory effect on Ψ even if EDE is gone (and it has not been 100% gone)

↳ Let's assume EDE is instantly gone followed by matter domination (not the case as there is still a lot of radiation during recombination)

In this case, Ψ_t & Ψ_t' are the boundary conditions with potentially large Ψ_t' sourced by EDE. Matter domination solution for Ψ is $\Psi = C_1 + C_2 (1+z)^{5/2}$. So match of boundary condition ⇒ decaying mode & $\dot{\Psi} = \dot{\Psi}_{\text{decaying}}$

So How to counter balance that? $\Omega_m h^2$?

- Well that also chang $D(z_*)$ so there is carefull balance and feebback here. But the main conclusion is: (1) EDE does not only change r_s. CMB also sees the potential evolution. so even $\delta\phi_{EDE}$ can be important!

Last part of the talk

- Why BAO affect Σm_ν (neutrinos) & <u>BAO-CMB tension</u> (from my research)

- <u>Neutrinos</u> → Neutrinos transition from relativistic to NR. When that transition happens? (1 massive ν)

$z_{nr} \sim 1890 \left(\frac{\Sigma m_\nu}{1eV}\right)$, Table

Σm_ν	z_{NR}
0.5	~945
0.1	~190

so $m_\nu \lesssim 0.5$ will impact the propagation of CMB photons (additional ISW)

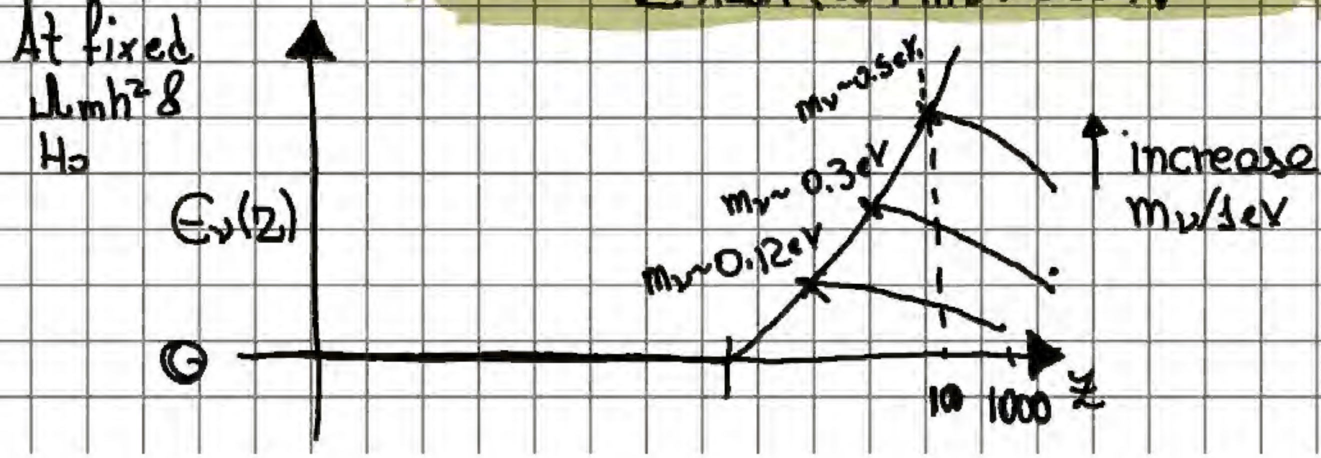
• $m_\nu \gg 0.5\,eV$ will change matter-radiation equality and that will affect driving force and also θ_s/θ_d ratio ("anything" that changes $H(z)$ will affect θ_s/θ_d ratio)

• for $m_\nu \ll 0.5\,eV$, it will affect $D(z_*)$ as

$$\begin{cases} z \ll z_{nR} \rightarrow \text{neutrinos redshift as } (1+z)^3 \\ z \gg z_{nR} \rightarrow \text{neutrinos redshift as } (1+z)^4 \end{cases}$$

At zero order BAO always see neutrinos as Dark matter. CMB does not as at fixed $\Omega_m h^2$ the swap between $\Omega_{cb} h^2$ and $\Omega_\nu h^2$ chandes $D(z_*)$

if we define $$\epsilon_\nu(z) \equiv \left(\frac{E_{\Lambda CDM}(z \mid m_\nu = 0.06\,eV)}{E_{\Lambda CDM}(z \mid m_\nu > 0.06\,eV)}\right)^2 - 1$$



from NR to R

Toy model: Instantaneous neutrino transition

↳ Assume neutrinos go from NR to R in a heartbeat at $z_{nR} = 1890\,(m_\nu/1\,eV)$. We can also use $\Omega_\nu h^2 \sim m_\nu/94.1\,eV$

· So $\rho_{nR}(z) = \begin{cases} \Omega_\nu^0 (1+z)^3 & \text{for } z \leq z_{nR} \\ \Omega_\nu^0 \dfrac{(1+z)^4}{(1+z_{nR})} & \text{for } z \geq z_{nR} \end{cases}$

The difference between the $z \geq z_{nR}$ and $\Omega_\nu^0 (1+z)^3$ evolution for $\delta z = z - z_{nR} \ll 1$ (but > 0) is

(1) $\dfrac{\Omega_\nu^0 (1+z)^4}{(1+z_{nR})} \underset{\text{Taylor expansion}}{\sim} \Omega_\nu^0 (1+z_{nR})^3 + \dfrac{4\Omega_\nu^0 (1+z)^3}{(1+z_{nR})}\Big|_{z=z_{nR}} \delta z$ relativistic

(2) $\Omega_\nu (1+z)^3 \sim \Omega_\nu^0 (1+z_{nR})^3 + 3\Omega_\nu^0 (1+z)^2\Big|_{z=z_{nR}} \delta z$ non-relativistic

So $\delta(H(z)/H_0) \sim (1) - (2) \approx \Omega_\nu^0 (1+z_{nR})^2 \delta z$

$$\Rightarrow \boxed{\left(\frac{h\,\delta(H/H_0)}{\delta z}\right)\Big|_{z_{nR}^+} \sim \left(\frac{m_\nu}{94.1\,eV}\right)\left(1 + 1890\,\frac{m_\nu}{1\,eV}\right)^2}$$

The derivative has a kink as the $z \to z_{nR}^-$ of $\delta(H/H_0) \to 0$

• How can we plot $\epsilon_\nu(z)$? z_{nR} for $m = 0.06\,eV$: $z_{nR} \sim 113$ (when the fiducial model has a kink in the derivative)

• so $z \lesssim 113$, $\epsilon_\nu = \left(\frac{E^2_{fid}}{E^2_{h}} - 1\right) = 0$, fid = fiducial

$h \to$ heavy $m_\nu > 0.06$

• at $z \sim 113$ first kink in E_{fid}. How to propagate to ϵ'?

$$\epsilon' = \frac{2E_f E_f'}{E_h^2} - \frac{2E_f^2}{E_h^3} E_h' \text{ . Only } E_f' \text{ jumps so...}$$

$$\boxed{\epsilon'_+ - \epsilon'_- = 2\left(\frac{E_f}{E_h^2}\right)\Bigg|_{z=113}\left(E'_{f+} - E'_{f-}\right)}$$

• then $113 \lesssim z < z_{NR}^{m_\nu}$ $\epsilon(z)$ grows fast

• At $z = z_{NR}^{m_\nu}$, another kink (now on E_h)

$$\epsilon'_+ - \epsilon'_- = -2\left(\frac{E_f^2}{E_h^3}\right)\Bigg|_{z=z_{NR}}\left(E'_{h+} - E'_{h-}\right)$$

• At $z > z_{NR}^{m_\nu}$, neutrino redshifts as $(1+z)^4$ in both models. What happens to the fractional difference?

↳ Let $c_i \equiv (\Omega_m h^2 - \Omega_\nu h^2)$ and $r_i = \Omega_R h^2 + \Omega_{\nu i} h^2/(1+z_{nR})$, $i = f, h$

Then: $h^2 E^2 \sim c_i (1+z)^3 + r_i (1+z)^4$. For $m_\nu > 0.06\,\mathrm{eV}$,

$c_h < c_f$ (f = fiducial $m_\nu = 0.06\,\mathrm{eV}$; h = heavy $m_\nu > 0.06\,\mathrm{eV}$

We can also assume that $\underbrace{r_f \sim r_h}_{\equiv r}$ because

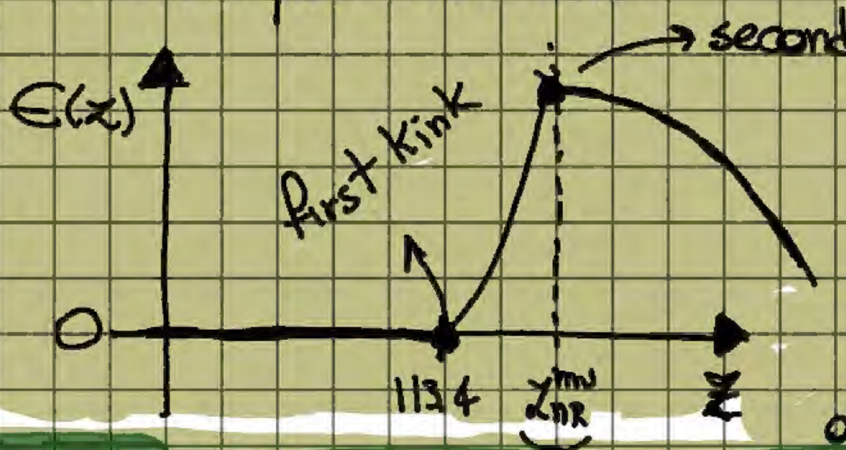
$$\frac{\Omega_\nu h^2}{(1+z_{nR})} \sim \boxed{\frac{m_\nu/94.1}{1890(m_\nu/1)} \text{ that is independent of } m_\nu}$$

$$E_f^2 - E_h^2 = \frac{(c_f - c_h)}{h^2}(1+z)^3 = \Delta\Omega_\nu h^2 (1+z)^3/h^2 \therefore$$

$$\epsilon = \frac{E_f^2}{E_h^2} - 1 \approx \frac{\Delta\Omega_\nu h^2 (1+z)^3}{c_h(1+z)^3 + r(1+z)^4} = \Delta\Omega_\nu h^2/(c_h + r(1+z))$$

$$\text{So } \epsilon = \frac{\Delta\Omega_\nu h^2}{c_h + cr(1+z)} \Rightarrow \boxed{\frac{d\epsilon}{dz} \sim -\frac{\Delta\Omega_\nu h^2 r}{(c_h + r(1+z))^2} < 0}$$

So the plot looks like



Remember that

$$\epsilon = \frac{E^2(0.06)}{E^2(m_\nu)} - 1$$

at fix $\Omega_m h^2$ and h

This plot has one big problem. We know now the physics of accoustic oscillations that fixes the amount of dark matter and baryons

So, from now on, $\Omega_{cB}h^2$ & $D(z_B)$ is fixed!

And for $m_\nu < 0.5eV$, the ammount of dark matter plus baryons is $\Omega_{cb}h^2 = \Omega_m h^2 - \Omega_\nu h^2$ (neutrinos are relativistic as $z_{NR} \sim 900$ for $m_\nu \sim 0.5eV$

So what should we fix is $\Omega_{cb}h^2$ not $\Omega_m h^2$. At fixed $\Omega_c h^2$, heavier neutrino mass implies higher $\Omega_m h^2$ (at low-z). BAO sees that, doesn't like it and force H_0 to readjust to keep $D(z \sim 0.5)$ fix.

One key thing here, even though we wrote in the beggining of this lecture that

$$r_s \sim 144.4 \left(\frac{\Omega_m h^2}{0.14}\right)^{-0.252} \left(\frac{\Omega_b h^2}{0.024}\right)^{-0.08}$$

for $m_\nu \ll 0.5eV$

but at $z > z_*$ neutrinos are relativistic so this $\Omega_m h^2$ actually is $\Omega_{cb}h^2$. If we dont change $T_{0\nu}$ and N_{eff} then $\partial r_s / \partial m_\nu \sim 0$!! (not exactly zero because $\rho_\nu \sim \rho_{\nu, rel}[1 + (m_\nu/T_\nu)^2]$ but this is small ($m_\nu \ll 0.5eV$)

So let's come back to our toy model where

$$\rho_\nu(z) = \Omega_\nu \times \begin{cases} (1+z)^3 & \text{if } z \leq z_{nr} \\ (1+z)^4/(1+z_{nr}) & \text{if } z \geq z_{nr} \end{cases}$$

and let's fix both $\Omega_{cb}h^2$ and $D(z=0.5)$. We will show this reduces H_0 (at $z<0.5$ neutrinos are NR)

$$\frac{H(z)^2}{100\,\text{km/s/Mpc}} = \Omega_m h^2 (1+z)^3 + \Omega_R h^2 (1+z)^4 + h^2 - \Omega_m h^2 - \Omega_R h^2$$

$$= h^2 + \Omega_m h^2 \left((1+z)^3 - 1\right) + \Omega_R h^2 \left((1+z)^4 - 1\right)$$

Let's call $Q(z) \equiv H^2(z)/100\,\text{km/s/Mpc}$. So,

$$\frac{D(z_{BAO})}{\text{Mpc}} = \frac{c\,[\text{km/s}]}{100\,\text{km/s/Mpc}} \int_0^{z_{BAO}} dz' / \sqrt{Q(z')}$$ and at an infinitesimal

$\delta\Omega_m h^2$ we want δh^2 so that $\delta D(z_{BAO}) \approx 0$

So $0 = \delta D = \frac{\partial D}{\partial h^2} \delta h^2 + \frac{\partial D}{\partial \Omega_m h^2} \delta \Omega_m h^2$ at z_{BAO}

$$\Rightarrow \delta h^2 = -\underbrace{\frac{\partial D/\partial \Omega_m h^2}{\partial D/\partial h^2}}_{\equiv \alpha} \delta \Omega_m h^2 \equiv -\alpha\, \delta \Omega_m h^2$$

It is not hard to see that

$$\frac{\partial D}{\partial h^2} = -\int_0^{z_{BAO}} \frac{dz'}{Q^{3/2}(z')} \quad \text{and} \quad \frac{\partial D}{\partial \Omega_m h^2} = -\int_0^{z_{BAO}} \frac{((1+z')^3-1)}{Q^{3/2}(z')}\, dz'$$

and both are negative numbers, so $\alpha > 0$

$\delta h^2 = -\alpha\, \delta \Omega_m h^2 < 0$ given that, as we increase m_ν, $\delta \Omega_m h^2 > 0$ at fixed $\delta \Omega_b h^2$

$\hookrightarrow \alpha_{BAO} \sim 0.87 \rightarrow m_\nu = \begin{cases} 0.06 \text{eV} \Rightarrow h \sim 0.674 \\ 0.5\text{eV} \Rightarrow h \sim 0.671 \end{cases}$

$$\boxed{\begin{array}{l} \delta h \sim -0.87 \dfrac{\Delta m_\nu}{94.6\,\text{eV}} \\ \delta h \sim -0.01\, \Delta m_\nu \end{array}}$$

$\hookrightarrow$ Now what is the sign of $E^2(z) = H^2(z)/H_0^2$?

$E^2 = Q(z)/h^2 \therefore \delta E^2 = \dfrac{\delta Q}{h^2} - \dfrac{Q}{h^4}\delta h^2$ and we know that

$\delta Q = \delta h^2 + [(1+z)^3 - 1]\delta \Omega_m h^2$ & $Q/h^4 = E^2/h^2$

$h^2 \delta E^2 = \delta h^2 (1 - E^2) + [(1+z)^3 - 1]\delta \Omega_m h^2$ & $\delta h^2 = -\alpha\, \delta \Omega_m h^2$

$\rightarrow h^2 \delta E^2 = \delta \Omega_m h^2 \left[(1+z)^3 - 1 + \alpha(E^2 - 1)\right]$ and that is > 0

so for the heavier m_ν, $E_h^2 > E_f^2$ (f = fiducial) so $\delta E^2 > 0$ at const $\Omega_c h^2$ and const $D(z_{BAO})$

Therefore we have $E_h^2 - E_f^2 > 0$, h = heavy ($m_\nu > 0.06$)

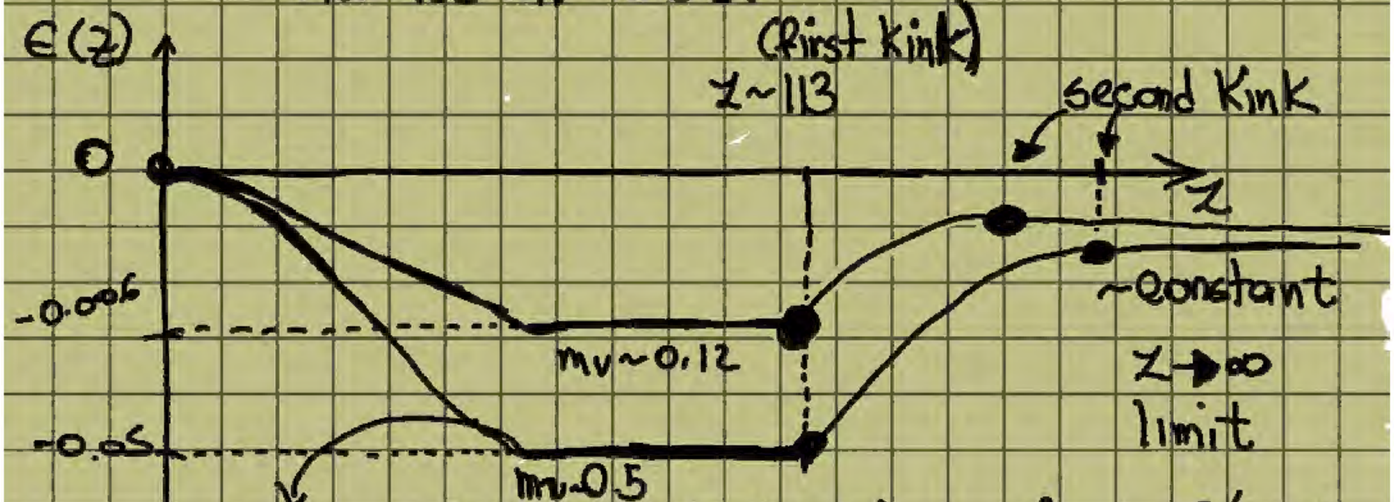
$$\rightarrow \boxed{\epsilon(z) = \frac{E_f^2}{E_h^2} - 1 < 0}$$

- in this toy model $\epsilon(0) = 0$ and $\epsilon(z \to 0^+) < 0$
h = heavy neutrino $m_\nu > 0.06$ eV

What is the behaviour $\epsilon(z \to \infty)$? Both neutrinos (heavy & fiducial) are relativistic. We have already showed that $r_i = \Omega_\nu h^2/(1+z_{nr}) \sim$ independent of $m_\nu \rightarrow H_f(z) \rightarrow H_h(z) \rightarrow \epsilon(z \to \infty) \rightarrow h_h^2/h_f^2 - 1 < 0$

so $\epsilon(z \to \infty) \rightarrow -0.02(m_\nu - 0.06)$

Plot for $m_\nu \sim 0.5$ eV



Plateau: $\frac{E_f^2(z)}{E_h^2(z)} \sim \left(\frac{h_h}{h_f}\right)^2 \left(\frac{1+f_f}{1+f_h}\right) - 1$, $f_i \equiv \Omega_{\nu i} h^2 / \Omega_{cb} h^2$

So the plateau before the first kink is

$$\epsilon \sim -0.02\Delta m_\nu - (\Delta m_\nu / 94.1\,\text{eV}) / \underbrace{\Omega_c h^2}_{\sim 0.12}$$

$$\epsilon \sim -0.02\Delta m_\nu - \Delta m_\nu / 11.292 \sim -0.1\Delta m_\nu$$

for $m_\nu \sim 0.5\,\text{eV}$ then $\epsilon \sim -0.05$ at this plateau

So neutrinos with $m_\nu > 0.06\,\text{eV}$ needs dark energy to maintain $D(z_*)$ fixed at fixed $D(z_{BAO})$ and fixed $\Omega_{cb}h^2$.

Let's investigate that:

Remember that $\epsilon(z) = \dfrac{E_f^2}{E_h^2} - 1 \rightarrow E_h^2 = E_f^2/(1+\epsilon(z))$

Define: $I(z) \equiv \displaystyle\int_0^z \frac{dz'}{E_f(z')}$ and $J(z) \equiv \displaystyle\int_0^z \frac{dz'(1-\epsilon(z'))^{1/2}}{E_f(z')}$. Then

- $D_h(z) = \dfrac{c}{H_h^0} J(z)$ and $D_f(z) = \dfrac{c}{H_f^0} I(z)$ $\left(\begin{array}{l} f \equiv \text{fiducial} \\ h = \text{heavier } m_\nu \end{array}\right)$
- Fixing BAO $D(z_B)$ means that $\dfrac{J(z_B)}{H_h^0} = \dfrac{I(z_B)}{H_f^0}$ $\therefore$

$H_h^0 = H_f^0 \left(\dfrac{J(z_B)}{I(z_B)}\right)$; ie $D(z_B)$ fixes the Hubble constant.

$$\cdot \text{ So } \frac{D_h(z_*)}{D_f(z_*)} = \overbrace{\frac{I(z_B)}{J(z_B)}}^{\text{ratio of Hubble constants}} \times \frac{J(z_*)}{I(z_*)} \quad \text{(ratio of Distance to recombination)}$$

$$\text{Now } J(z_*) = \int_0^{z_*} \frac{dz'}{E(z')}\sqrt{1+\epsilon(z')} \sim \int_0^{z_*} \frac{dz'}{E(z')} \times (1+\epsilon(z')/2) \text{ for } \epsilon \ll 1$$

$$\cdot \text{ So } J(z_*) \sim I(z_*) + \frac{1}{2}\int_0^{z_*} dz' \frac{\epsilon(z')}{E_f(z')}.$$

$$\cdot \text{ Define } \langle\epsilon\rangle_z \equiv \int_0^z dz' \frac{\epsilon(z')}{E_f(z')} \Big/ \int_0^z \frac{dz'}{E_f(z')}, \quad f = \text{fiducial model}$$

$$\text{Then } \frac{D_h(z_*)}{D_f(z_*)} \simeq \frac{(1+\langle\epsilon\rangle_*/2)}{(1+\langle\epsilon\rangle_B/2)} \simeq 1 + \frac{1}{2}\left(\langle\epsilon\rangle_* - \langle\epsilon\rangle_B\right)$$

$$\rightarrow \Delta D/D \sim \left(\langle\epsilon\rangle_* - \langle\epsilon\rangle_B\right)/2. \text{ Let's define:}$$

$$I_{B*} \equiv \int_{z_B}^{z_*} \frac{dz'}{E_f(z')} \rightarrow \underline{I_* = I_B + I_{B*}} \quad \left(\text{Notation } I_x = \int_0^x \frac{dz'}{E(z')}\right)$$

$$\text{Now } \langle\epsilon\rangle_* = \frac{1}{I_*}\left[\int_0^{z_B} + \int_{z_B}^{z} \frac{dz'\,\epsilon(z')}{E_f(z')}\right] = \frac{I_B}{I_*}\langle\epsilon\rangle_B + \frac{I_{B*}}{I_*}\langle\epsilon\rangle_{B*}$$

Therefore

$$2\frac{\Delta D}{D} \approx \langle\epsilon\rangle_B \underbrace{\left(\frac{I_B}{I_*} - 1\right)}_{= -I_{B*}/I_*} + \frac{I_{B*}}{I_*}\langle\epsilon\rangle_{B*}. \text{ So, finally}$$

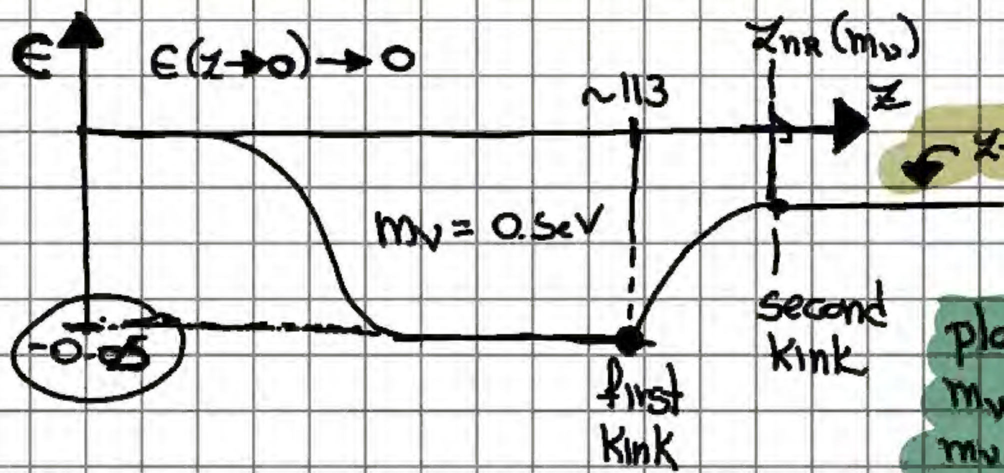
this is evaluated in LCDM to first order!

$$\Rightarrow \frac{\Delta D_*}{D_*} \approx \frac{I_{B*}}{2 I_*}\left[\langle \epsilon \rangle_{B*} - \langle \epsilon \rangle_{B} \right]$$

this is the average from z_{BAO} to z_*

this is the average from $z=0$ to z_B

But we know the shape of $\epsilon(z)$



ratio of h^2 $\leftarrow z \to \infty$

plateau
$m_\nu = 0.5 \to -0.05$
$m_\nu = 0.12 \to -0.005$

It is clear that for $z_B = 0.5$, $\langle \epsilon \rangle_B \sim 0$, while $\langle \epsilon \rangle_{B*}$ is quite negative!!

so $\boxed{\Delta D_* / D_* < 0}$, in νLCDM

Now we need dark energy to make $\epsilon(z)$ positive to drive $(\langle \epsilon \rangle_{B*} - \langle \epsilon \rangle_B) \sim 0$!!!

• $\epsilon > 0$ requires phantom dark energy

Review $w_0 w_a$: $w(z) = w_0 + w_a z/(1+z)$

Let's define $X(z) \equiv \frac{\rho_{DE}(z)}{\rho_{DE,0}} = \exp\left[3\int_0^z d\ln(1+z')(1+w(z'))\right]$

So let's define $\boxed{\epsilon(z) = E^2_\Lambda(z)/E^2_w(z) - 1}$

define $B(z) = \Omega_m h^2[(1+z)^3 - 1] + \Omega_{r,\nu} h^2[(1+z)^4 - 1]$

So $E^2_w(z) = 1 + B(z)/h^2_w + \Omega_{DE,w}[X(z) - 1]$ (flat model)

for ΛCDM, $E^2_\Lambda(z) = 1 + B(z)/h^2_\Lambda$. (same $B(z)$ as we are only changing DE)

Therefore,

$$\underbrace{E^2_w(z) - E^2_\Lambda(z)}_{\text{Needs to be negative}} = B(z)\left[\frac{1}{h^2_w} - \frac{1}{h^2_\Lambda}\right] + \Omega_{DE,w}[X(z) - 1]$$

case 1: $(h_w^{-2} - h_\Lambda^{-2}) < 0$; case 2: $(h_w^{-2} - h_\Lambda^{-2}) > 0$

Remember that phantom $w < -1$ X(z) decreases

↳ If case 1, it helps. If case 2 absolutelly necessary.

So let's implement BAO normalization with expansion around LCDM

$w(z) = -1 + \delta w(z)\,;\ X(z) = -1 + \delta X(z)$ (Toy model)

$$\delta X(z) = \exp\left[3\int_0^z d\ln(1+z')\,(1+(-1+\delta w)\right] - 1$$

$$= \exp\left[3\int_0^z d\ln(1+z')\,\delta w\right] - 1$$. Now δw is small (expand the exp)

then $$\boxed{\delta X(z) \approx 3\int_0^z d\ln(1+z')\,\delta w(z')}$$

Now We need to change both δh and δX to fix BAO

$$Q(z) \equiv \frac{H^2(z)}{100\,\mathrm{Km/s/Mpc}} = \Omega_m h^2(1+z)^3 + \Omega_R h^2(1+z)^4 + (h^2 - \Omega_m h^2 - \Omega_R h^2)X(z)$$

$$X(z) = \underbrace{-1}_{\mathrm{LCDM}} + \delta X\ ,\quad h = h + \delta h$$

$$\Rightarrow \delta Q = 2h_\Lambda^2\,\delta\ln h\,\underbrace{X(z)}_{=1}\Big|_{\mathrm{LCDM}} + (h^2 - \Omega_m h^2 - \Omega_R h^2)\,\delta X(z)$$

Now $E^2 = Q/h^2$; $\delta E^2 = \delta Q/h_\Lambda^2 - E_\Lambda^2\,\delta(h^2)/h^2$

$$\Rightarrow \delta E^2 = 2\,\delta\ln h + \overbrace{\frac{(h^2 - \Omega_m h^2 - \Omega_R h^2)}{h^2}}^{\Omega_\Lambda}\delta X(z) - 2E_\Lambda^2\,\delta\ln h$$

$$\Rightarrow \delta E^2 = -2\left[E_\Lambda^2(z) - 1\right]\delta\ln h + \Omega_\Lambda\,\delta X(z)$$

Finally, $\epsilon(z) = E_\Lambda^2/E_w^2 - 1$, $E_w^2 \approx E_\Lambda^2 + \delta E^2$

and $\frac{E_w^2}{E_\Lambda^2} \approx -2\left(\frac{E_\Lambda^2-1}{E_\Lambda^2}\right)\delta\ln h + \frac{\Omega_\Lambda}{E_\Lambda^2}\delta X(z) + 1$

$$\Rightarrow \epsilon(z) = 2\left(\frac{E_\Lambda^2-1}{E_\Lambda^2}\right)\delta\ln h - \frac{\Omega_\Lambda}{E_\Lambda^2}\delta X(z)$$

Now, fix $D_M(z_B) = \left[\frac{c\,[\mathrm{km/s}]}{100\,\mathrm{km/s/Mpc}}\right] \times \frac{1}{h} \times \int_0^z dz' / \underbrace{E(z)}_{\sqrt{E_\Lambda^2+\delta E^2}}$

$\approx \int_0^{z_B} \frac{dz'}{E_\Lambda^2\sqrt{1+\delta E^2/E_\Lambda^2}}$

$\approx \int_0^{z_B} \frac{dz'}{E_\Lambda^3}\left(1 - \frac{1}{2}\frac{\delta E^2}{E_\Lambda^2}\right)$

(linear order in δE^2)

Lets define $I_B = \int_0^{z_B} dz' / E_\Lambda(z)$, so

$$\frac{\delta D_M}{0} \approx -\delta\ln h - \frac{1}{2I_B}\int_0^{z_B} \frac{dz' \times \delta E^2(z')}{E_\Lambda^3(z')}$$

$$\Rightarrow \delta\ln h = \frac{1}{2I_B}\int_0^{z_B} \frac{dz'}{E_\Lambda^3(z')}\delta E^2$$

We know that $\delta E^2 = -2[E_\Lambda^2 - 1]\delta\ln h + \Omega_\Lambda \delta X(z)$

so $2I_B\delta\ln h = -2\delta\ln h\left[\int_0^{z_B}\frac{dz'}{E_\Lambda} - \int_0^{z_B}\frac{dz}{E_\Lambda^3}\right] + \Omega_\Lambda\int_0^{z_B} dz' \frac{\delta X}{E^3}$

$$\Rightarrow \delta\ln h = \frac{\Omega_\Lambda}{2} \times \int_0^z \frac{dz'\,\delta X(z')}{E^3(z')} \Big/ \int_0^z dz'/E^3(z') \equiv \frac{\Omega_\Lambda}{2}\langle\delta X\rangle_B$$

$$\boxed{\delta\ln h = -\frac{\Omega_\Lambda}{2}\langle\delta X\rangle_B}$$

Coming back to $\epsilon(z)$ we get

$$\epsilon(z) = 2\left(\frac{E_\Lambda^2 - 1}{E_\Lambda^2}\right)\left(\frac{-\Omega_\Lambda}{2}\langle \delta X\rangle_B\right) - \frac{\Omega_\Lambda}{E_\Lambda^2}\delta X$$

$$\Rightarrow \epsilon(z) = \frac{-\Omega_\Lambda}{E^2(z)}\left[\delta X(z) + (E_\Lambda^2 - 1)\langle \delta X\rangle_B\right]$$

→ here we show something quite important

WE NEED PHANTOM BEHAVIOUR

if $\delta w > 0$ (ie, $w > -1$), $\delta X > 0$ and $\langle \delta X\rangle_B > 0 \Rightarrow \epsilon < 0$

And remember that neutrinos induce $\epsilon < 0$.

We need $\epsilon > 0$ at some point in the range $0 < z < z_*$ so that $D_M(z_*)$ is not affected

Neutrinos need phantom if $D(z_B)$ & $D(z_*)$ are compatible with LCDM with $m_\nu = 0.06\,eV$

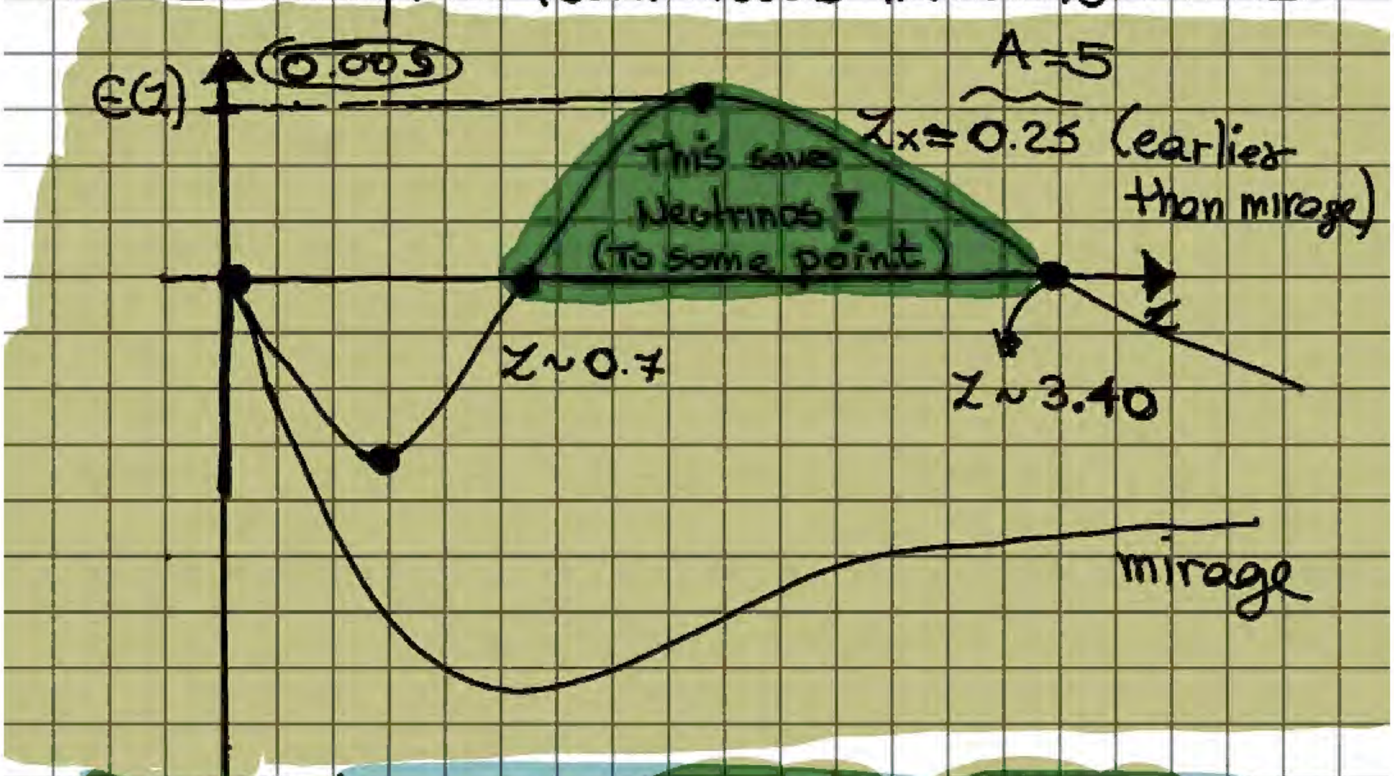
→ But there is more. The fact BAO really like mirage models is quite bad news to neutrinos → just allowing phantom is not enought

Let's remember $w_a = -A(1+w_0)$ and $z_x = 1/(A-1)$

At mirage $z_x \sim 0.37$ and mirage balance low and high redshift behavior so $\Delta H_0 = 0$ (predicted by CMB and low-z BAO)

↳ that balance is not what we want!

If we plot (both models have $w_0 = -0.95$



Two questions left: How much does this saves neutrino? How analytically we can estimate the zero crossings

• What is the magnitude of the effect?

↳ (LCDM) Fiducial model: $\bar{h} = 0.685$, $\bar{\Omega}_m = 0.315$

$\bar{\Omega}_R = 9\times10^{-5}$, $\Omega_\Lambda \approx 0.685 \Rightarrow \Omega_m h^2 \approx 0.143$

In our toy model $z_B = 0.5$ and $z_* = 1100$ fixed to the values predicted by the toy model!

↳ The main result of our toy approximation is that a difference in ϵ averages determine

$$\frac{\Delta D_*}{D_*} \sim \frac{1}{2}\frac{I_{B*}}{I_*}\left(\langle\epsilon\rangle_{B*} - \langle\epsilon\rangle_B\right) \quad \text{(previous result)}$$

with $I_{B*} = \int_{z_B}^{z_*} \frac{dz'}{\hat{E}(z')}$, $I_B = \int_0^{z_B} \frac{dz'}{\hat{E}(z')}$, $\langle\epsilon\rangle_{B*} = \frac{1}{I_{B*}}\int_{z_B}^{z_*} dz' \frac{\epsilon(z')}{\hat{E}(z')}$

and $\langle\epsilon\rangle_B = (1/I_B)\int_0^{z_B} dz' \frac{\epsilon(z')}{\hat{E}(z')}$ and $I_* = \int_0^{z_*} dz'/\hat{E}(z')$

To first order, the I's are evaluated in LCDM. With 3 point simpson rule you can estimate I_B & 4 point SR in $\ln(1+z)$ we can estimate I_{B*} & I_* well. For neutrinos we can easily calculate using approximate ϵ that

$(\sum m_\nu, \Delta D_*/D_*) \sim (0.12, -1.6\times10^{-3})$

$(0.3, -6.3\times10^{-3})$

for $w_0 = -0.95$, $A = 5$, $w_a = -0.25$

we have $\Delta D_* / D_* \sim 2.3 \times 10^{-3}$ $(\sim 0.23\%)$

Planck as we saw it has $\sigma(\ln D_*) = \underbrace{\frac{1}{4} \sigma \ln \omega_m h^2}_{\sim 0.3\%}$

So $\delta(\ln D_*) \sim 0.7\, \sigma_{Planck}$ from dark energy

and around $+2\,\sigma_{planck}$ for $m_\nu \sim 0.3\,eV$

(almost linear)

$\hookrightarrow$ So in $w_0 w_a$ you could have gone to .3 eV

In ΛCDM only to $\sim 0.15\,eV$

this assuming tolerance of ~ 1 sigma

Now let's show an approximate estimation of $\epsilon = 0$ roots

Remember that $\epsilon(z) = -\frac{\Omega_\Lambda}{E^2(z)} \left[\delta X(z) + (E_\Lambda^2 - 1) \langle \delta X \rangle_2 \right]$

and $\ln X(z) = 3 \int_0^z d\ln(1+z)\, (1 + w(z))$, $w(z) = w_0 + w_a \frac{z}{1+z}$

and $w_a = -A(1+w_0)$ & $z_x = 1/(A-1)$ & $w(z_x) = -1$

So $1+w(z) = 1+w_0 - \frac{A(1+w_0)z}{1+z} = \overbrace{(1+w_0)}^{\equiv d}\left[1-\frac{Az}{1+z}\right]$:

$\frac{(1+w(z))}{d} = \frac{(1-(A-1)z)}{(1+z)}$. Therefore:

$$\ln X(z) = 3d\int_0^z dz' \underbrace{\frac{(1-(A-1)z)}{(1+z)^2}} = 3d\int_0^z dz' \left[\frac{\cancel{1}+A-\cancel{1}-(A-1)(1+z')}{(1+z')^2}\right]$$

$$\frac{C_1}{(1+z')} + \frac{C_2}{(1+z')^2} = \underset{-(A-1)}{C_1}(1+z') + C_2 = A$$

$$\text{So } \ln X(z) = 3d\int_0^z dz'\left[\underbrace{\frac{A}{(1+z')^2}} + \frac{(1-A)}{(1+z')}\right]$$

$$\underbrace{-\frac{A}{(1+z)}\Big|_0^z} \qquad (1-A)\ln(1+z)$$

$$\frac{-A}{(1+z)} + A = \frac{Az}{1+z}$$

$$\text{Finally } \ln X(z) = 3d\underbrace{\left\{(1-A)\ln(1+z)+\frac{Az}{1+z}\right\}}_{\text{Define this as } F_A(z)} \equiv 3d\,F(z)$$

$$X(z) = e^{3dF(z)} \rightarrow X_w(z) - \underbrace{X_{LCDM}}_{=1} = e^{3dF(z)} - 1$$

assuming $\delta X \ll 1$, $\delta X \approx 3d\,F_A(z)$, $d=(1+w_0)$

So $\delta X \approx 3d\, F_A(z)$, $F_A(z) = (1-A)\ln(1+z) + \frac{Az}{1+z}$

First question. We know $w(z)$ cross -1 at $z_* = 1/A - 1$

When $\delta X = 0$ (ie, density cross. Here remember that $w_0 = -1$. So initially $\rho_w > \rho_\Lambda$. Then after w becomes phantom, ρ starts decreasing. So $\delta X(z_{X=1}) = 0$ should happen at $z_{X=1} > z_*$ → to avoid confusion I will call this $z_{w=-1}$ from now on

For $A = 5$ (our model), $z_{w=-1} = 0.25$

and $z_{X=1}$ is $\approx$ $\underbrace{\ln(1+z_{X=1})}_{\substack{0.47\\(z\approx 0.6)}} = \frac{5}{4}\underbrace{\left(\frac{z_{X=1}}{1+z_{X=1}}\right)}_{\approx 0.375}$ $z \approx 0.6$

and $1.25 \times 0.375 \approx 0.4687$

so the solution is $z_{X=1} \approx 0.6$ (at $z > 0.6$ $\delta X < 0$)

The more difficult part is to estimate $\langle \delta X \rangle_B$

$$= \int_0^{z_B} dz'\, \delta X / E_\Lambda^3(z) \Big/ \int_0^{z_B} \frac{dz'}{E_\Lambda^3(z)}$$ to first order.

$z_B = 0.5$. Ignoring radiation

$$E_\Lambda^2(z) \approx 1 + \Omega_m\left[(1+z)^3 - 1\right] \approx 1 + \Omega_m\left(3z + 3z^2 + z^3\right)$$

$$\rightarrow \boxed{E^3(z) = \left[1 + \Omega_m\left(3z + 3z^2 + z^3\right)\right]^{3/2}}$$

So we need to compute

$\int_0^{z_B} dz' \frac{F_A(z')}{E^3(z')}$ and $\int_0^{z_B} \frac{dz'}{E^3(z')}$. I will use Simpson 1/3 rule (3 points)

$$\begin{cases} z = 0 & F_A(z) = 0,\ E^{-3} = 1 \quad (A=5) \\ z = 0.25 & F_A(z) \approx 0.107,\ E^{-3} \approx 0.674 \quad (A=5) \\ z = 0.5 & F_A(z) \approx 0.045,\ E^{-3} \approx 0.433 \quad (A=5) \end{cases}$$

Simpson 1/3 rule: $\int_a^b dx\, y(x) = \frac{(b-a)}{3}\left(y_0 + 4y_1 + y_2\right)$

$$\text{So } \langle F_A \rangle_{z_B} \approx \frac{4(0.674)\times(0.107) + 0.045\times 0.433}{1 + 4\times 0.674 + 0.433}$$

$$\approx (0.288 + 0.019)/(1 + 2.696 + 0.433)$$

$$\approx \frac{1}{4.129} \times 0.307 \approx 0.0743$$

So $\langle F_A \rangle_{z_B} \approx 0.0743$ (that has ~5% error)

So now let's come back to $\epsilon(z)$

$$\epsilon(z) = -\frac{\Omega_\Lambda}{E^2_\Lambda(z)}\left[\delta X(z) + (E^2_\Lambda - 1)\langle \delta X\rangle_B\right]$$

with $\delta X_B = 3d\, F_A(z)$; $F_A(z) = (1-A)\ln(1+z) + Az/(1+z)$.

For $A = 5$: (1ϵ, $z_* = 0.25$)

$$\begin{cases} F_A(z) = -4\ln(1+z) + \frac{5z}{1+z} \\ \langle F_A\rangle_{z_B} \approx 0.0743 \end{cases}$$

We also have $(E^2_\Lambda - 1) \approx \Omega_m(3z + 3z^2 + z^3)$

So we need to solve the equation

$$-4\ln(1+z) + \frac{5z}{1+z} + 0.0743\underbrace{\Omega_m}_{\text{fiducial model} = 0.315}[3z + 3z^2 + z^3] = 0$$

table (z : LHS)

0.5	$\rightarrow$ 0.1
1.0	$\rightarrow$ −0.1
2.0	$\rightarrow$ −0.45
3.0	$\rightarrow$ −0.32
4.0	$\rightarrow$ 0.46

so we can see that my plot with $z_1 \sim 0.7$ and $z_2 \sim 3.4$ was OK !

For mirage, $A = 2.618$ and the equation is

$$-1.61\ln(1+z) + \frac{2.6178z}{(1+z)} + 0.13\,\Omega_m(3z + 3z^2 + z^3) = 0$$

for mirage the LHS is always positive

$z \to$ LHS

$0.5 \to 0.31$
$1.0 \to 0.47$
$2.0 \to 1.03$
$3.0 \to 2.30$
$4.0 \to 4.58$

so no $\epsilon(z)$ crossing zero on mirage.

$\hookrightarrow$ Physics conclusion: we need $w_0 w_a$ model with $w(z)$ crossing -1 at $z_{w=-1} \ll 0.5$ to make $\sigma(m_\nu)$ less constraining than ΛCDM

The last topic I want to talk about is to ask "what is up with τ_{reio} (optical depth of of reionization and the DESI-CMB tension"*

* Weiner (2603.18131) prefers to think as the tension between CMB Θ_* constraint and what you get from CMB accoustic peak structure (excluding peak location)

The high z behaviour of w_0w_a & the optical depth of reionization

- w_0w_a CPL parameterization $w(z) = w_0 + \frac{w_a z}{(1+z)}$ is not the "linear" (in scale factor as $z/(1+z) = (1-a)$) in a Taylor series of a perturbative controlled expansion. If it were, we would need some argument on why the higher order terms are negligible when $z \gtrsim 1$ (e.g. some symmetry).

↳ what w_0w_a is a phenomenological fit that works reasonable well some simple models

↳ w_0w_a has a particular way of balacing low-z and high-z dark energy induced changes that CMB (with fixed $D(z_*)$ and $\Omega_m h^2$) is quite sensitive to. For example, if we ask what is the fractional change w_0w_a induces on $\int_2^{z_*} dz/E(z)$ compared to LCDM, it is ~5% (and that is huge)

0812.1794

In fact, Linder showed for the first time (around 2008) that if we try to construct a set of principle Components, valid on a range $0 < z < z_{max}$ where z_{max} is high enough (he considered $z_{max} = 9$!) to check what are the shapes CMB+SN can actually constrain, then the treatment of the $z > z_{max}$ range (e.g. fix $w = -1$ or marginalize over $w = w_\infty$) affect the results (ie, the shape of the PCs or the signal-to-noise SNR ($\alpha_i / \sigma_{\alpha_i}$) of PC amplitudes α_i)

↳ in Linder argument, α_i is the magnitude of the PC_i that we must measure with high SNR to be able to tell apart interesting models. For exemple he showed that for simple scalar field models with Freezing/thawing behavious, 97%+ of SNR_{total} are in the first 2 PCs (SNR_{total} = SNR of all PCs)

On the other hand, something more complicated (like $w(a) = -1 + A[1 - \cos(\ln a)]$) needs 3PCs to reach SNR ~92% of SNR_{total}. How many PCs you want to marginalize over depends on the physical prior you have on the behaviour of $w(a)$. But the main point is that fix vs marginalize $w(z>9)$ affected SNR of the first 3PCs quite considerably. the high redshift behaviour matters.

↳ In my paper with students Liu & Zhu and Professor Hu, we investigated how axions could generate the phantom mirage w/o coupling dark matter and dark energy (arxiv: 2510.14957). Our baseline axion model has $\log_{10}(m_a) = -32.5$, $f_a \equiv \frac{\Omega_a h^2}{\Omega_m h^2} = 0.086$ (the dark energy component is Λ). the $\Delta\chi^2 \sim -7.5$ here the $\Delta\chi^2$ number excluded $\ell^{EE}_{\ell<32}$ from Planck (but in the fit, $C_{\ell<32}^{EE}$ was included). That is worse than the $\Delta\chi^2 \simeq -18.5$ improvement provided by WoWa. However, when $C_{\ell<32}^{EE}$ was excluded from the fit, the same axion model improved the fit compared to ΛCDM by $\Delta\chi^2 \sim -14.5$ (much better!) In this case, the optical depth of reionization $\tau \simeq 0.102$ (when $C_{\ell<32}^{EE}$ is present, $\tau \sim 0.06$))

$\hookrightarrow$ So, what is going on?

· What θ_* fixes? Answer: it is not $\Omega_m h^2$ (we need peaks/lensing to fix $\Omega_m h^2$). Key is the fact that θ_* does not fix $D_M(z_*)$ but $D_M(z_*)/r_d$ and that fixes $(\Omega_m h^{3.4})$. We already discussed that τ increases from ~ 0.06 to ~ 0.09 reduces $\Omega_c h^2$ by 2.8%. (that is $\sim 2\sigma$ given the $\sim 1.2\%$ Planck error bars)

So why $\Delta\tau$ induced decreased in $\omega_m h^2$ was relevant? Because we saw that in ΛCDM, there is a tension between CMB+SN vs CMB+SN+BAO on $D(z=0.8)/r_d$. (CMB+SN: $D(z=0.8)/r_d \sim 1.5\%$ higher than CMB+SN+BAO). This quantity is in the z-range where BAO data is the most constraining (so we can interpret CMB+SN predicting what CMB+SN+BAO measures)

↳ fixing $D(z\approx0.8)/r_d$ at fix D_*/r_d involves a redistribution between $z \ll 0.8$ & $z \gg 0.8$ expansion that phantom w_0w_a does well (and axions do not have that). But again, we are relying on $z \gg 1$ behaviour of the w_0w_a model. Again, from a theoretical perspective, that can be seen as "bad", "risky", "unwise".

↳ but $\Delta \ell_m$ decreasing by $\approx 2.8\%$ with Θ_* fix imply $\Delta(D_M/r_d)$ down by $\sim 1.7\%$ (we will show that)

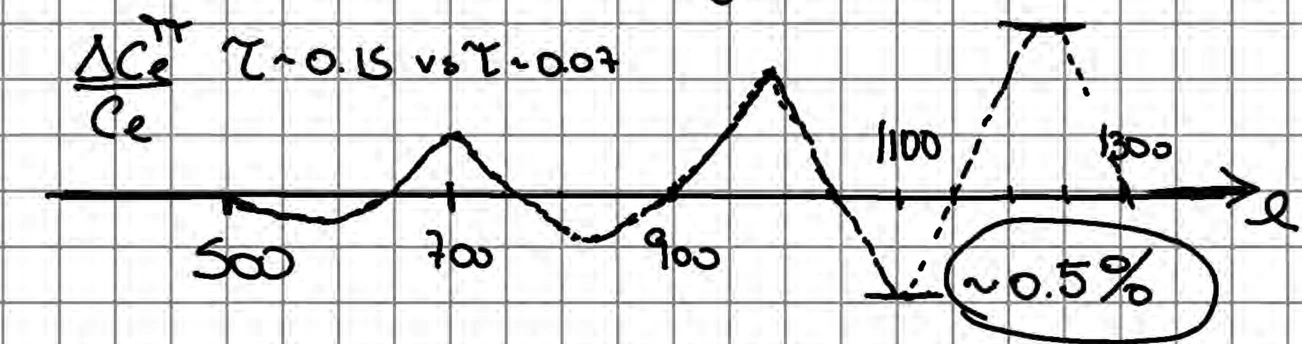
↳ this changes the preferred dark energy model.

Solution: $w_0 w_a$ or Something like + axions ?
τ

- How CMB measures τ (excluding higher order statistics besides lensing)

Two ways: first C_ℓ^{TT} measures $A_s e^{-2\tau}$ really well.

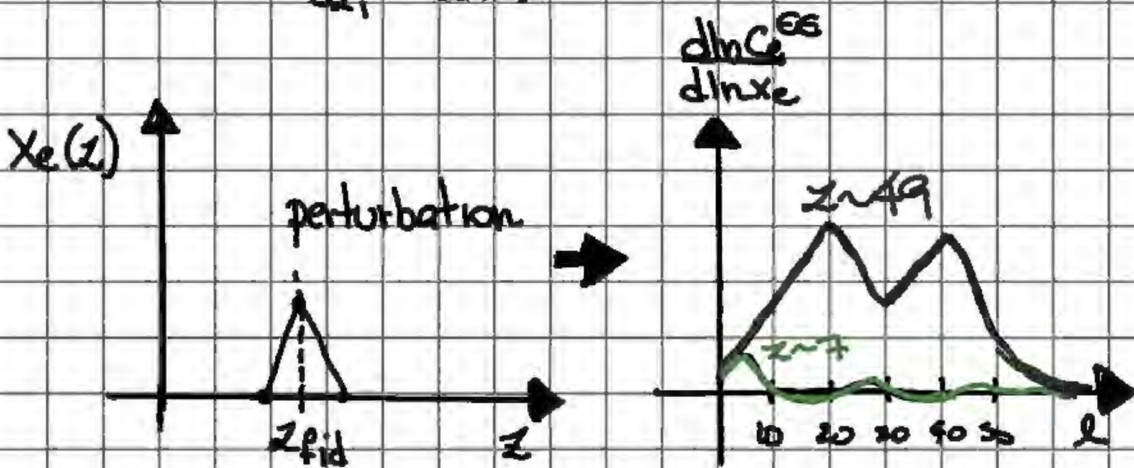
At fixed parameters besides A_s and τ, more A_s → more structure formation → more lensing. Lensing can be measured by 4-point function $C_L^{\phi\phi}$ or by smothing of C_ℓ^{TT} peaks in the damping tail.



There is an excess smothing that induces $\tau \sim 0.09$

Another way to measure τ is via $C_{\ell\lesssim 30}^{EE}$

$$\tau(z_1, z_2) \propto \int_{z_1}^{z_2} dz \, \frac{(1+z)^2}{E(z)} \overbrace{x_e(z)}^{\text{ionization fraction}}$$



There are 2 instruments on Planck (LFI vs HFI)

→ LFI preferred $\tau\sim 0.09$ w/ high $\tau(z\gtrsim 15)$ contribution
HFI prefers low $\tau\sim 0.056$ w/ little $\tau(z\gtrsim 15)$

This affects $\Omega_c h^2$ which affects not only dark energy but also constraints on $\sum m_\nu$ (2504.21813)

What is missing? How changes in inflation can be used to change τ (2606.20795)

→ Tanisha, Wayne Hu & V. Mirande

Thank you very much ! (maybe I continue writing later)