

New Developments in Birational Geometry

A Three-Day School and Research Workshop

Preparatory Meeting for the 2027 Summer School

Simons Center for Geometry and Physics (SCGP)

October 7–9, 2026

Overview

Birational geometry has recently acquired a new and unexpected source of invariants. Rather than being extracted from the geometry of a variety directly, these invariants are read off from its *quantum* cohomology and from the Hodge-theoretic structures attached to it. The workshop is devoted to these developments, to the techniques that make them computable, and to the questions they open.

The meeting will take place at the Simons Center for Geometry and Physics (SCGP), Stony Brook, New York, from October 7 to October 9, 2026. Two mini-courses, of two lectures each, run from Wednesday morning through Thursday morning and build the necessary background from the ground up. The research talks begin on Thursday at 11:00 and continue through Friday.

The organizing theme is the theory of *atoms*, introduced by Katzarkov, Kontsevich, Pantev, and Yu [11], in which a canonical decomposition of the quantum cohomology of a smooth projective variety produces new birational invariants. The construction draws on classical and non-commutative Hodge theory [10] together with genus-zero Gromov–Witten theory, and relies in an essential way on the decomposition theorem for the quantum D -module of a blowup due to Iritani [9] and on the decomposition and framing theory for F -bundles developed in [8]. Equivariant extensions of the formalism, obtained by combining atoms with the modular symbols of Kontsevich, Pestun, and Tschinkel, are developed in [2]. Applications include obstructions to rationality and to linearizability of group actions that are invisible to older techniques.

Each mini-course lecture lasts one hour and forty minutes; each research talk lasts fifty minutes. All times indicated below are local times at the Simons Center (Eastern Time, New York).

Who Should Attend

The workshop is open to everyone working on birational geometry and on Hodge theory, classical or non-commutative, regardless of the route by which they arrived there. No single background is assumed, and the mini-courses are built on the premise that participants will come from different directions and will not all know the same things.

In particular, we hope to see:

- **Birational geometers** — working on rationality and stable rationality, decomposition of the diagonal, equivariant and G -equivariant birational geometry, Fano and Calabi–Yau varieties, or the birational geometry of moduli spaces. The new invariants are birational invariants first; the quantum-cohomological machinery is a means of computing them.
- **Hodge theorists** — whether the objects of interest are variations of Hodge structure, period maps and degenerations, and Hodge loci, or the non-commutative side: ncHodge structures, Hodge structures on the Hochschild homology of a category, and their limiting behavior. Both appear here, and the passage between them is one of the points of the theory.
- **Symplectic geometers and mirror symmetry specialists** — Fukaya categories, Landau–Ginzburg models, SYZ and family Floer theory, non-Archimedean and tropical approaches to mirror symmetry.
- **Algebraic geometers and category theorists** — semiorthogonal decompositions, derived categories, Kuznetsov components, and the categorical approaches to rationality.
- **Mathematical and theoretical physicists** — see below.

Graduate students and postdoctoral researchers are especially welcome. Familiarity with basic algebraic geometry and cohomology is desirable; previous exposure to Gromov–Witten theory or mirror symmetry will be useful but is not required, and the first mini-course starts from the definitions.

Connections with Physics

Much of the structure discussed at this workshop is familiar to physicists under other names, and participants from theoretical and mathematical physics should find the material closer to home than the title suggests.

The quantum cohomology of a variety is the chiral ring of the associated A -twisted sigma model; the Frobenius manifold structure and the WDVV equations governing it are the two-dimensional topological field theory of Dubrovin’s lectures [4]. The spectral data of multiplication by the Euler vector field, which controls the decomposition into atoms, is the eigenvalue spectrum familiar from tt^* geometry and the classification of $N = 2$ supersymmetric theories in the sense of Cecotti and Vafa [3]. The non-commutative Hodge structures of [10] are a mathematical formulation of the tt^* and topological-string data of Landau–Ginzburg and sigma models. Convergence of Gromov–Witten potentials, treated here through non-Archimedean analytic geometry, is the question of whether the instanton sum defining the quantum product actually makes sense beyond formal power series.

Areas of overlap of particular interest include topological string theory and the topological B -model, $N = 2$ supersymmetric gauge theories and their Coulomb branches, integrable

systems and isomonodromic deformations, BPS spectra and wall-crossing, and homological mirror symmetry. Physicists are warmly encouraged to attend, and the lecturers have been asked to make the dictionary between the two vocabularies explicit wherever it arises.

Aims

Among the main objectives of the workshop are:

- to explain what the new quantum-cohomological invariants of birational geometry are, and what they can and cannot detect;
- to introduce the basic geometric and enumerative structures entering the theory of atoms, starting from genus-zero Gromov–Witten theory;
- to explain how Gromov–Witten theory gives rise to Frobenius-type structures and to the A-model F -bundle;
- to introduce the non-Archimedean framework used in the study of convergence and analytic continuation of the relevant structures;
- to explain decomposition phenomena for F -bundles and their role in the definition of Hodge atoms;
- to present explicit examples, with particular emphasis on flag varieties and related homogeneous spaces;
- to make explicit the dictionary with the corresponding structures in two-dimensional field theory and topological string theory;
- to discuss current applications, ongoing research, and open problems; and
- to prepare the ground for the 2027 summer school.

For background on Gromov–Witten theory, Frobenius manifolds, F -manifolds, equivariant localization, and Berkovich geometry, see, for example, [13, 5, 6, 4, 7, 1]. For the theory of atoms itself, the basic references are [11, 2]; see also [10, 9, 8]. For the physical origins of the tt^* structures, see [3].

Relation to the 2027 Summer School

This workshop is conceived as a *preparatory meeting* for the summer school to be held in 2027. Its purpose is twofold: to assemble, in a compact and self-contained form, the prerequisites that participants of the summer school might be assumed to have; and to allow the organizers and lecturers to test and calibrate the structure of the extended program.

Accordingly, the two mini-courses are designed as a first pass through the material that will be developed at greater length and in greater generality in 2027.

Participants who intend to take part in the 2027 summer school are therefore particularly encouraged to attend. More details on the summer school, including dates, venue, and the full list of lecturers, will be announced in due course.

Schedule at a Glance

Wednesday, Oct. 7	Thursday, Oct. 8	Friday, Oct. 9
10:00–11:40 Mini-course I.1	09:30–11:10 Mini-course II.2	09:30–10:20 Kontsevich
13:30–15:10 Mini-course I.2	11:20–12:10 Pantev	10:30–11:20 Harder
16:20–18:00 Mini-course II.1		13:00–13:50 Pomerleano
		13:55–14:45 Cavenaghi
		14:50–15:40 Cai
		16:20–17:10 Katzarkov

The period from 15:45 to 16:20 is kept free of sessions on all three days.

Schedule

Day 1: Wednesday, October 7, 2026

Time	Activity
10:00–11:40	Mini-course I, Lecture 1 — P. A. Muniz Martins
11:40–13:30	Lunch break
13:30–15:10	Mini-course I, Lecture 2 — P. A. Muniz Martins
15:10–16:20	Break
16:20–18:00	Mini-course II, Lecture 1 — L. F. Cavenaghi

Mini-course I lecturer: Pedro Antonio Muniz Martins (Universidade Estadual de Campinas, Unicamp)

10:00–11:40 Lecture 1: Introduction to Gromov–Witten Invariants and Localization

The first lecture introduces the basic ideas of Gromov–Witten theory, with an emphasis on the genus-zero theory relevant to quantum cohomology. We discuss moduli spaces of stable maps, evaluation morphisms, curve classes, and the geometric meaning of Gromov–Witten invariants. No prior acquaintance with the subject is assumed.

The second part of the lecture introduces equivariant cohomology and the localization principle. Particular attention will be paid to torus actions and to the localization techniques that make explicit Gromov–Witten computations possible. The general formalism

will then be specialized to flag varieties, where the geometry of torus-fixed points and invariant curves provides a natural combinatorial framework for calculations.

The lecture concludes with a computational demonstration illustrating how Gromov–Witten invariants of complete intersections in flag varieties can be approached algorithmically.

Standard references for Gromov–Witten theory and localization include [13, 12, 6, 5].

13:30–15:10 Lecture 2: Frobenius Manifolds, Euler Vector Fields, and Materialization

The second lecture introduces Frobenius manifolds and the geometric structures naturally associated with genus-zero Gromov–Witten theory (equivalently, the structure of a two-dimensional topological field theory). Beginning with the WDVV equations and the quantum product, we discuss the compatibility between multiplication, metric, and flat structures that underlies the definition of a Frobenius manifold.

Particular attention will be given to Euler vector fields and the additional grading information they encode. We will examine multiplication by the Euler vector field, its spectral behavior, and the role played by the corresponding eigenvalue data in the geometry of the theory. This spectral data is what will later separate the atoms, and is the same data that organizes the tt^* picture of $N = 2$ theories [3].

The lecture then turns to the materialization procedure and explains how the formal structures arising from quantum cohomology can be related to analytic geometric objects. This provides one of the bridges between Gromov–Witten theory, Frobenius geometry, and the structures that later enter the definition of atoms [11].

For the classical theory of Frobenius manifolds and its relation to two-dimensional topological field theory, see [4]; for the closely related theory of F -manifolds, see [7].

Mini-course II lecturer: Leonardo F. Cavenaghi (International Center for Mathematical Sciences – Sofia, ICMS-Sofia)

16:20–18:00 Lecture 1: Berkovich Geometry and the A-Model F -Bundle

This lecture introduces the non-Archimedean geometric framework used in the construction of the A-model objects appearing in the theory of atoms. We begin with the basic notions of non-Archimedean analytic geometry and Berkovich analytic spaces, emphasizing those features that are relevant to families arising from quantum cohomology. A standard reference for the foundations of this theory is [1].

We then discuss how this analytic framework can be used to formulate and study convergence properties of Gromov–Witten potentials. Particular attention will be given to the transition from formal quantum-cohomological data to analytic families, as in [11].

The second part of the lecture introduces F -manifolds and F -bundles. After reviewing the integrability condition for the multiplication on the tangent sheaf, we construct the A-model F -bundle and describe the geometric information encoded by its multiplication, connection, and spectral data [7, 8]. The relation with the genus-zero Gromov–Witten theory developed earlier in the day will be made explicit throughout the discussion.

Day 2: Thursday, October 8, 2026

Time	Activity
09:30–11:10	Mini-course II, Lecture 2 — L. F. Cavenaghi
11:20–12:10	Tony Pantev (University of Pennsylvania)

09:30–11:10 Mini-course II, Lecture 2: Decomposition Theorems for F -Bundles and Hodge Atoms

Leonardo F. Cavenaghi (ICMS-Sofia)

The second lecture develops the structural results for F -bundles that lead to the notion of atoms. We first discuss local and global decomposition phenomena and the role of spectral data in separating the different components of the corresponding geometric structure [8]. The blowup decomposition of the quantum D -module [9] will be presented as the main source of birational control.

We then introduce the Hodge-theoretic locus and explain the passage from the spectral decomposition of the A-model F -bundle to the definition of *Hodge atoms*, following [11]. The discussion will emphasize both the geometric motivation for the construction and the information retained by the resulting components. Examples will be used to illustrate how atoms can encode birational information that is not visible at the level of ordinary cohomological invariants, and how they obstruct rationality and linearizability.

This lecture closes the preparatory part of the program and leads directly into the research talks, which open half an hour later.

From 11:20 Research Talks

The research part of the workshop opens on Thursday late morning and continues through Friday. The talks are devoted to recent developments in birational geometry, the theory of atoms, mirror symmetry, Hodge-theoretic structures, and related aspects of algebraic and complex geometry. They are intended to connect the foundational material of the mini-courses with current research questions and applications, and to outline the program of the 2027 summer school.

Day 3: Friday, October 9, 2026

Time	Activity
09:30–10:20	Maxim Kontsevich (IHÉS)
10:30–11:20	Andrew Harder (Lehigh University)
11:20–13:00	Lunch break
13:00–13:50	Daniel Pomerleano (University of Massachusetts Boston)
13:55–14:45	Leonardo F. Cavenaghi (ICMS-Sofia)
14:50–15:40	Jiaji Cai (Stony Brook University)
15:40–16:20	Break
16:20–17:10	Ludmil Katzarkov (SCGP)

Speakers

- **Maxim Kontsevich***, Institut des Hautes Études Scientifiques (IHÉS);
- **Tony Pantev***, University of Pennsylvania;
- **Andrew Harder**, Lehigh University;
- **Daniel Pomerleano**, University of Massachusetts Boston;
- **Ludmil Katzarkov**, Stony Brook University and the Simons Center for Geometry and Physics;
- **Leonardo F. Cavenaghi**, ICMS-Sofia;
- **Jiaji Cai**, Stony Brook University.

* remote talk

Titles and abstracts of the research talks will be circulated closer to the meeting.

References

References

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- [12] M. Kontsevich, *Enumeration of Rational Curves via Torus Actions*, in R. Dijkgraaf, C. Faber, and G. van der Geer (eds.), *The Moduli Space of Curves*, Progress in Mathematics, Vol. 129, Birkhäuser, Boston, 1995, pp. 335–368.
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